Design of High Accuracy Tracking Systems with H_∞ Preview Control

Antonio Moran Cardenas, Javier G. Rázuri, Isis Bonet, Rahim Rahmani, and David Sundgren

***Abstract*—Positioning and tracking control systems are an important component of autonomous robot applications. This paper presents the design method of tracking control systems based on H_∞ preview control where the present and future desired positions of the robot are used to determine the control actions to be applied so that the robot describes the desired trajectory as close as possible. The performance improvements achieved with H_∞ preview control have been examined in the frequency and time domains for different types of reference signals when applied to a one-dimensional positioning system. It was found that preview control improves the tracking performance by improving the phase response of the tracking system.**

***Index Terms*—Robotics, planning and scheduling, predictive control, H_∞ control, tracking control, control H_2, frequency-domain analysis, time-domain analysis.**

I. Introduction

PRECISE positioning/tracking control is being studied in many manufacturing fields in order to improve the accuracy and performance of manufacturing process and manipulator driving systems which every time demand more precise, robust and efficient control systems [1]. Certain behavior is desired in positioning/tracking control systems: fast response and convergence, zero tracking error and robustness against changes in the system itself and/or its environment.

The classical way to solve the tracking control problem for linear time-invariant systems has been to design a one-degree-of-freedom, or better, a two-degrees-of-freedom controller which will achieve the desired performance as close as possible. The inherent shortcoming of the classical approach is the overdesign that is entitled in requiring a performance that is independent of the specific measurable signal to be tracked. In the case of standard H_∞ control, technique introduced by Zames [2] a controller is designed for a worst case reference signal which may be different from the one which is actually encountered. Obviously such an approach may introduce a considerable emphasis on the tracking properties of the system which may lead, in some cases, to insufficient disturbance attenuation and robustness properties of the closed-loop system [3].

In order to avoid the shortcomings of classical control when the reference signal is known a priori (for some interval of time), the concept of preview tracking control has been introduced. According to this control strategy, the control input is calculated using not only information corresponding to the present state of the system but also using the known future value of the reference signal. The advantages of preview control has been studied by several authors [4]–[6]. An updated review of preview control is presented in [7]. The most of preview control methods are based on classical LQ/LQG control theory where preview control is used for further reduction of the minimum value achievable by the cost function being optimized [8]–[10].

However, LQG control methods present the limitation of how to include robustness specifications for designing the controller which are usually defined in terms of H_∞ norms. Several preview control methods based on H_∞ control theory, which take into acount the unknown disturbances to design control systems with better performance measures, have been developed. In [11]–[13], the preview performance in terms of H_∞ criterion was investigated. Some authors have been presented solutions for discret-time cases [14], [15], and others for continuous-time problems [16]–[18].

This paper presents the solution to the H_∞ preview tracking control problem. The analytical formulation of the H_∞ preview tracking control law has been derived in the continuous time domain. Preview control laws have been derived for two tracking problems: (a) the reference signal is perfectly known in advance for the total working time and (b) the reference signal is previewed for a fixed interval of time. For both cases the structure of the preview controller corresponds to a two-degrees-of-freedom controller with feedback and feedforward (preview) parts. The benefits and limitations of the proposed control strategies are analyzed theoretically and experimentally in the time and frequency domains for a wide range of operating conditions and different types of reference signals.

This paper is structured as follows: Section II presents the structure of the experimental positioning/tracking system, while Section III describes the State Equation Model of the system. In Section IV, we described the formulation process related to the generalized plant. Experimental results and discussion are presented in Section V, while Section VI presents some conclusions of the research.

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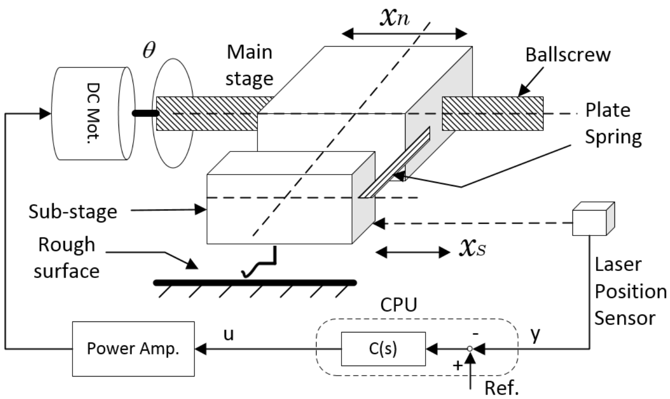

Fig. 1. Experimental tracking system

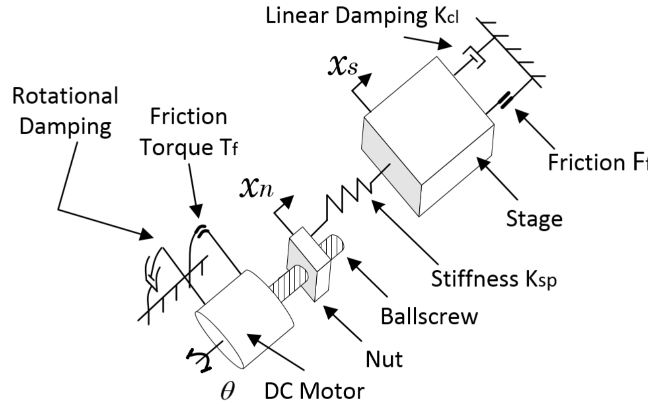

Fig. 2. Vibrational model of experimental system

II. Experimental System

A. Experimental System

Figure 1 shows the structure of the experimental positioning/tracking system. A DC motor rotates a ballscrew to longitudinally move a positioning stage which is composed of a main stage and a sub-stage connected through two flexible plate springs. The control objective is to place the sub-stage in an arbitrary desired position or to follow any desired trajectory. To do that, a laser position sensor measures the position of the sub-stage and sends the measured signal to a computer which calculates a control voltage according to the algorithm of the H_∞ controller. Using a power amplifier, the control signal is applied to the DC motor to control the rotational motion of the ballscrew in order to achieve the desired motion of the sub-stage. The DC motor and main stage are affected by friction torque and friction forces, respectively. The sub-stage moves on a rough surface whose degree of roughness (friction) may be varied to analyze the robustness of the system against external friction forces.

B. Vibrational Model

Figure 2 shows the equivalent vibrational model of the positioning/tracking mechanism. The flexible plates are modeled as a linear spring with stiffness K_{sp}. The rotational damping of the DC motor is represented by K_{cr} and the friction torque by T_f. External forces actuating on the sub-stage are represented by linear damping K_{cl} and friction force F_f.

III. State Equation Model

A. Nominal Model

Neglecting the inductance of the DC motor, its dynamics can be described by the following equation:

$$\left(\frac{R_a}{K_t}\right) T = K_{pa}u - K_e\dot{\theta}, \tag{1}$$

where T and u are the torque and voltage of the DC motor and $\dot{\theta}$ is the angular velocity of the ballscrew. The rotational motion equation of the driving mechanism (motor shaft and ballscrew) is given by

$$T - T_f - T_{qr} = J\ddot{\theta} + K_{cr}\dot{\theta}, \tag{2}$$

where T_f is the Coulomb friction torque and T_{qr} is an equivalent torque representing the rotational effect of the inertia of the main stage and is equal to

$$T_{qr} = K_{sp}K_{bs}(x_m - x_s), \tag{3}$$

where x_m and x_s represent the position of the main stage and sub-stage, respectively. The linear motion equation of the sub-stage is given by

$$(x_m - x_s)K_{sp} - F_f = M\ddot{x}_s + K_{cl}\dot{x}_s, \tag{4}$$

where F_f is the friction force affecting the sub-stage and M is the mass of the sub-stage. Defining the state vector:

$$\bar{x} = \left[\dot{\theta}, \theta, \dot{x}_s, x_s\right]^T, \tag{5}$$

and combining Equations 1 to 4, the following state-space equation of the nominal system P can be formulated:

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}_1\bar{w} + \bar{B}_2u, \tag{6}$$

where $\bar{w} = [T_f, F_f]^T$. The parameters specifications are shown in Table I.

TABLE I
List of Parameters of Experimental Model

Name	Simbol	Value
Power amplifier gain	K_{pa}	10
DC motor resistance	R_a	1.1 ohm
DC motor torque constant	K_t	0.0573 $N.m/A$
Back electromotive force constant	K_e	5.665 x 10^{-2} $v.s$
Moment of inertia	J	4.326 x 10^{-5} $kg.m^2$
Rotational damping factor	K_{cr}	4.550 x 10^{-3} $N.m.s$
Ballscrew transducing coefficient	K_{bs}	1.509 x 10^{-3} m/rad
Spring stiffness	K_{sp}	264 N/m
Linear damping factor	K_{cl}	0.747 $N.s/m$
Mass of sub-stage	M	0.244 kg

B. Generalized Plant with Integral Action

In order to design a controller with integral action to eliminate steady-state tracking errors, the generalized plant used for designing the controller is structured considering integral action. To do that, Equation 6 representing the nominal plant, is integrated with the equation of the tracking error e given by the expression:

$$e = r - x_s, \tag{7}$$

where r is the reference signal to be tracked. Defining the state vector

$$x = \left[\dot{\theta}, \theta, \dot{x}_s, x_s \int (r - x_s)dt\right]^T, \tag{8}$$

the state-space equation of the generalized plant with integral action is:

$$\dot{x} = Ax + B_{1p}\bar{w} + B_2u + B_r r, \tag{9}$$

where it can be noted that the reference signal r is not included in the disturbance vector $\hat{w}$ Matrices A, B_{1p}, B_2 and B_r are obtained by combining Equations 6 and 7.

IV. H$_\infty$ Preview Control Law

As it is well known, the first step for designing an H$_\infty$ controller is the formulation of the generalized plant. By defining the controlled output vector z as

$$z = \left[\rho_1\dot{x}_s, \rho_2 x_s, \rho_3 \int (r - x_s)dt, u\right]^T, \tag{10}$$

and the measured output vector y as:

$$y = \begin{bmatrix} x_s + \eta_1 \\ \int (r - x_s)dt + \eta_2 \end{bmatrix}, \tag{11}$$

the equations describing the generalized plant P are:

$$\dot{x} = Ax + B_1w + B_2u + B_r r \tag{12}$$

$$z = C_1x + D_{12}u \tag{13}$$

$$y = C_2x + D_{21}w \tag{14}$$

where the disturbance vector $w = [T_f, F_f, \eta_1, \eta_2]^T$ and $B_1 = \left[B_{1p}^T 0\right]^T$. Factors ρ_1, ρ_2 and ρ_3 in Equation 10 are weighting coefficients selected to tailor the performance and robustness specifications. η_1 and η_2 in Equation 11 represent measurement noise which are assumed to be independent of the plant disturbances $\bar{w}$. Given this assumption, matrices B_1 and D_{21} turn to be orthogonal. Also, given the components of the controlled output z, matrices C_1 and D_12 are orthogonal. The structure of the generalized plant is shown in Figure 3 (to gain clarity, the controlled output z is not shown).

According to H$_\infty$ control theory, given the generalized plant P, a controller is designed so that the H$_\infty$ norm of the transfer function T_{zw} from disturbances w to controlled output z is less than a given scalar γ:

$$\|T_{zw}\| < \gamma. \tag{15}$$

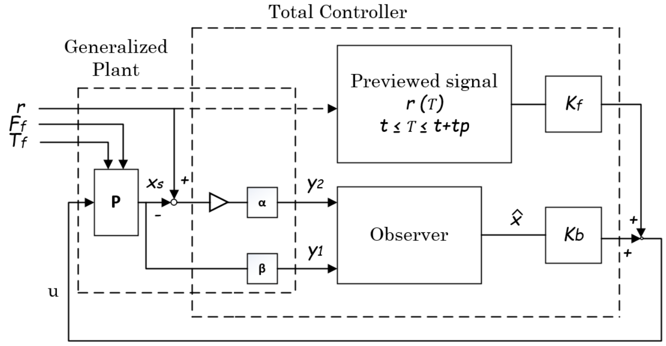

Fig. 3. Block diagram of generalized plant and preview controller.

Optimal H$_\infty$ controllers are designed for the minimum value of γ and H$_2$ controllers are designed for $\gamma = \infty$. By analogy with Game Theory, the H$_x$ control problem can be expressed as the minimization of the following cost function:

$$J = \int \left[z^Tz - \gamma_2 w^Tw\right] dt, \tag{16}$$

which can be expanded as:

$$J = \int_0^\infty \left[x^TC_1^TC_1x + \left[w^Tu^T\right] \begin{bmatrix} -\gamma^2 & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}\right] dt, \tag{17}$$

where $R = D_{12}^TD_{12}$. The conventional approach to design H$_\infty$ tracking systems has been to formulate a generalized plant which includes the reference signal as external disturbance and design a one or two-degrees-of-freedom controller which will achieve the desired specifications. The inherent shortcoming of this approach is the overdesign that is entitled when designing a controller independently of the reference signal to be tracked which is known a priori.

$$\hat{w} = [T_f, F_f, \eta_1, \eta_2, r]^T.$$

In other words, a controller is designed for the worst reference signal r_{worst} which is, for the most of cases, different to the desired reference signal r. To overcome the shortcoming of the classical H$_\infty$ approach, this paper proposes H$_\infty$ preview control where the known reference signal is used, as it is, for designing the controller. The design of the H$_\infty$ preview controller has been divided in two parts: first, H$_\infty$ preview control laws have been derived for the case of state feedback, and afterwards an observer was designed considering the conventional approach to design H$_\infty$ output feedback controllers. The structure of the H$_\infty$ preview controller is shown in Figure 3 where it can be noted the feedback part K_f and preview part K_p of the controller. H$_\infty$ control laws have been derived for two cases: (1) the reference signal is known for the total working time and (2) the reference signal is previewed for a fixed interval of time shorter than the total working time.

A. Reference signal known for the total working time

When the reference signal is known for the total working time $[t, t+T]$, the H_∞ preview controller is designed considering the generalized plant of Equations 12 and 13 and the following cost function:

$$J = \int_t^{t+T} \left[z^T z - \gamma_2 w^T w \right] dt \tag{18}$$

Using the Hamiltonian approach of Optimal Control Theory [19], the preview control law which minimizes the cost function of Equation 18 subject to the constraint of Equation 12 is given by:

$$u(t) = -R^{-1} B_2^T \left[P(t)x(t) + q(t) \right], \tag{19}$$

where $P(t)$ is the solution of the following differential Riccati equation:

$$\begin{aligned} -\dot{P(\tau)} = P(\tau)A - P(\tau)(B_2 R^{-1} B_2^T - \gamma^{-2} B_1 B_1^T)P(\tau) + \\ + A^T P(t) + C_1^T C_1, \end{aligned} \tag{20}$$

with $t \leq r \leq t+T$ and $P(\tau)$ satisfying the terminal constraint:

$$P(t+T) = 0;$$

$q(t)$ in Equation 12 is the solution of the following differential equation:

$$\begin{aligned} -\dot{q(t)} = (A^T - P(\tau)(B_2 - R^{-1} B_2^T - \gamma^{-2} B_1 B_1^T))q(\tau) + \\ + P B_r r(\tau), \end{aligned} \tag{21}$$

satisfying the terminal constraint:

$$q(t+T) = 0.$$

Since to compute $q(t)$ and $P(t)$, Equations 20 and 21 should be integrated backwards in time, the computational amount could turn cumbersome and impractical so that an approximated simplification easy to compute is necessary. Assuming the working time T is long enough so that $\dot{P)} = 0$ and $P = const$, $q(t)$ can be calculated from the following equation:

$$\begin{aligned} q(t) = exp\left[-(A_{cl}^T + M)t \right] \mathrm{x} \\ \mathrm{x} \int_t^{t+T} \left[exp\left[(A_{cl}^T + M)\tau \right] P B_r r(\tau) \right] d\tau, \end{aligned} \tag{22}$$

where

$$A_{cl} = A - B_2 R^{-1} B_2^T P \tag{23}$$

and

$$M = \gamma^{-2} B_1 B_1^T P. \tag{24}$$

A_{cl} in Equation 23 represents the state matrix of the closed-loop system for the case of only feedback control. It is important to note that the control law of Equation 19 has the structure of two-degrees-of-freedom controllers with feedback and feedforward (preview) parts so that preview control improves the tracking performance of the system without affecting its feedback characteristics (stability, disturbance attenuation, etc.).

B. Reference signal previewed for a fixed interval of time

In this case it is assumed the reference signal is known for a fixed interval of time T which is much shorter than the total working time assumed to be long enough. In this case, the cost function to be optimized is

$$J = \int_t^{\infty} \left[z^T z - \gamma^2 w^T w \right] d\tau, \tag{25}$$

which can be decomposed in two parts:

$$\begin{aligned} J = \int_t^{t+T} \left[z^T z - \gamma^2 w^T w \right] d\tau + \\ + \int_{t+T}^{\infty} \left[z^T z - \gamma^2 \hat{w}^T \hat{w} \right] d\tau, \end{aligned} \tag{26}$$

where

$$J_1 = \int_t^{t+T} \left[z^T z - \gamma^2 w^T w \right] d\tau \text{ and } J_2 = \\ = \int_{t+T}^{\infty} \left[z^T z - \gamma^2 \hat{w}^T \hat{w} \right] d\tau$$

Since the reference signal r is only known for the interval $[t, t+T]$, it is not included as a component of the disturbance vector w for this interval (cost function J_1), but it is included in $w = \hat{w}$ for the interval $[t+T, \infty]$. Since in this interval r is not known it should be replaced by r_{worst} as in standard H_∞ tracking control. From Linear Optimal Control Theory it is well known that the minimum value of the cost function J_2 is

$$J_2 = x^T(t+T)\hat{P}x(t+T), \tag{27}$$

where $\hat{P}$ is the solution of the following algebraic Riccati equation

$$\begin{aligned} \hat{P}A - \hat{P}(B_2 R^{-1} B_2^T - \gamma^{-2}(B_1 B_1^T + B_r B_r^T))\hat{P} + \\ + A^T \hat{P} + C_1^T C_1 = 0 \end{aligned} \tag{28}$$

From Equations 26 and 27, the cost function J can be expressed as:

$$J = x(t+T)\hat{P}x(t+T) + \int_t^{t+T} \left[z^T z - \gamma^2 w^T w \right] dt \tag{29}$$

Since the cost functions of Equations 29 and 18 are similar with the only difference being the term $x(t+T)\hat{P}x(t+T)$ in Equation 29, the control law which minimizes the cost function J of Equation 29 subject to the constraint of Equation 12, is also given by Equation 19 with $P(t)$ calculated from Equation 20 and $q(t)$ calculated from Equation 21. The only difference is the terminal value of P which for this case is

$$P(t+T) = \hat{P},$$

where $\hat{P}$ is already known from the Riccati equation 28. Since the computation of $q(t)$ is cumbersome when P varies with time, an approximation for the easy computation of q is necessary. Similarly as in subsection IV-A, assuming that $\dot{P} = 0$ then $P = const$ and takes the value for time $t+T$, e.g., $P = \hat{P}$. With this simplification, the control signal u can be computed easily at every stage of control.

V. Results and Discussion

In order to evaluate the tracking performance of the H_∞ preview controller, the response of the system has been examined in the time and frequency domains for different types of reference signals. The response of the system with H_∞ preview control has been compared with the response for H_∞ feedback controller with 1.5 degrees-of-freedom (DOF) and H_2 preview controller [20]. The H_∞ preview controller has been designed for the case when the reference signal is previewed for a fixed interval of time shorter than the total working time which is assumed to be long enough (sub-section IV-B). The preview time is chosen to be 1 second since longer times do not yield significant performance improvements.

A. Frequency Response

In order to compute the frequency response of the system with preview control, the Laplace transform $Q(s)$ of the preview part $q(t)$ of the control law of Equation 19 is required to be known. Using Equation 22 and recalling the definition of Laplace transform, where $R(s)$ is the Laplace transform of the reference signal $r(t)$ and $Q(s)$ is calculated as:

$$Q(s) = \left[Q_1(s) - Q_2(s)\right] R(s), \tag{30}$$

$$Q_1(s) = exp\left[Ts\right]\left[sI + A_{cl}^T + M\right]^{-1} \times exp\left[(A_{cl}^T + M)T\right] PB_r, \tag{31}$$

$$Q_2(s) = \left[sI + A_{cl}^T + M\right]^{-1} PB_r. \tag{32}$$

Figure 4 shows the gain and phase of the frequency response of the tracking system for H_2 preview control (solid line) and 1.5DOF H_2 feedback control (broken line). It can be noted that although the gain of the response is higher at high frequencies for 1.5DOF H_2 feedback control, the phase of the response for H_2 preview control is zero for frequencies up to 10 Hz. Zero phase response is desired in tracking systems since it is always desirable that the system responds to the reference input without delays. Figure 5 compares the gain and phase of the frequency response for H_∞ preview control (solid line) and 1.5DOF H_∞ feedback control (broken line). Similarly as for H_2 control, the gain of the response is higher for 1.5DOF H_∞ feedback control but the phase of the response for H_∞ preview control is close to zero for frequencies up to 1 Hz and is positive for the medium frequency range up to 10 Hz. This positive phase characteristics indicate that the system with H_∞ preview control responds in advance to the reference input. Since in the medium frequency range the gain of the frequency response for H_∞ preview control is low, the controller tries to compensate this deficiency by responding in advance with positive phase. Figure 6 shows the gain and phase of the frequency response for H_∞ preview control (solid line) and H_2 preview control (broken line). It can be noted that the gain of the response is higher for H_∞ preview control than for H_2 preview control. The phase response shows the zero-phase characteristics of H_2 preview control and the positive characteristics of H_∞ preview control discussed before.

B. Time Response

The time response of the tracking system with H_∞ preview control has been analyzed for sinusoidal and step reference inputs.

a) **Sinusoidal Response**
Figure 7 shows the time response of the system for a sinusoidal reference input of 0.5 Hz. Figure 7 (a) corresponds to 1.5DOF H_∞ feedback control and Figure 7 (b) to H_∞ preview control. In Figure 7 (a) it is noted that although the amplitude of the response of the system (solid line) is almost the same as the reference input (broken line), the system responds with delay and is unable to track the reference input with zero error. In Figure 7 (b) it can be noted that the sinusoidal reference input and the response for H_∞ preview control practically overlap from the initial time. It is clear that the tracking error is almost zero for the total working time.

b) **Step Response**
It is usually said that preview control improves the tracking performance of the system especially in situations where the signal to be tracked varies with time, and no significant improvement can be achieved for step reference signals for situations when the control action starts just when the reference signal is applied. However and as it is shown in Figure 8, it has been found that although the step response for the system with 1.5DOF H_∞ feedback control (broken line) and H_∞ preview control (solid line) are almost the same, Figure 8 (a), the control signal u is different for both controllers, Figure 8 (b). Preview control demands lower control signals and therefore requires less control energy to achieve the same tracking performance than 1.5DOF H_∞ feedback control. Calculated results show that the control energy can be reduced even by 10% when using H_∞ preview control.

VI. Conclusions

This paper has presented a novel design method of positioning/tracking systems based on H_∞ preview control using the known future value of the reference input. Analysis of the frequency response shows that preview control improves the tracking performance by improving the phase response of the tracking system so that the system responds to the reference input without delay. The step response of the tracking system shows that H_∞ preview control requires less control energy than 1.5DOF H_∞ feedback control to achieve the same positioning performance.

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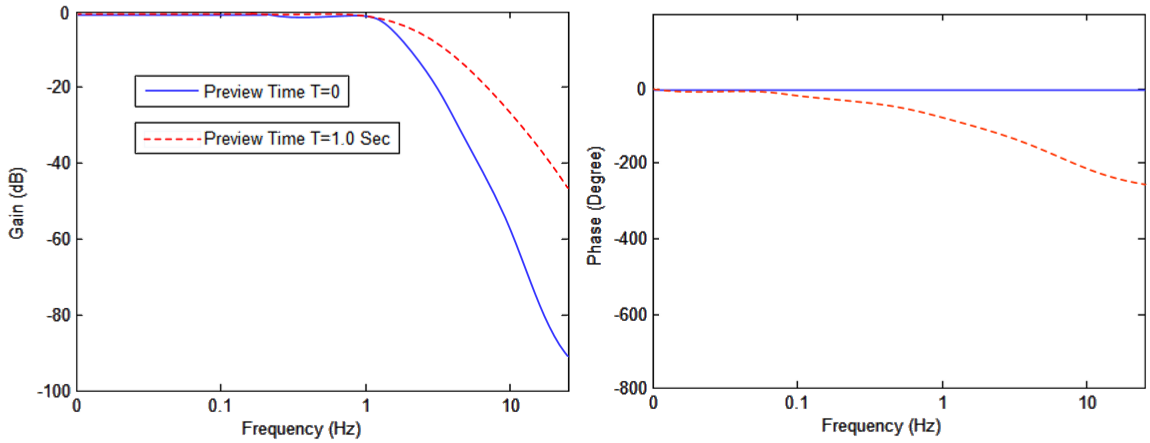

Fig. 4. Frequency response of H_2 preview control

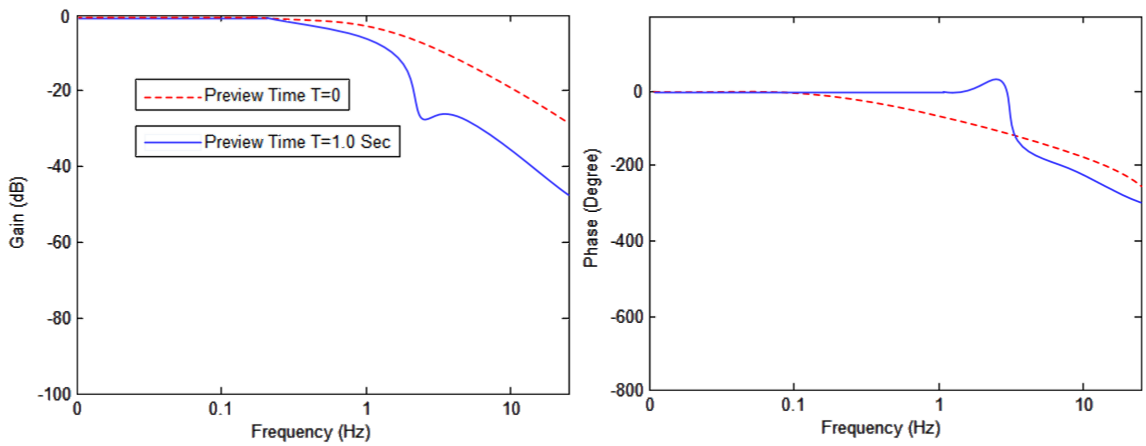

Fig. 5. Frequency response of H_∞ preview control

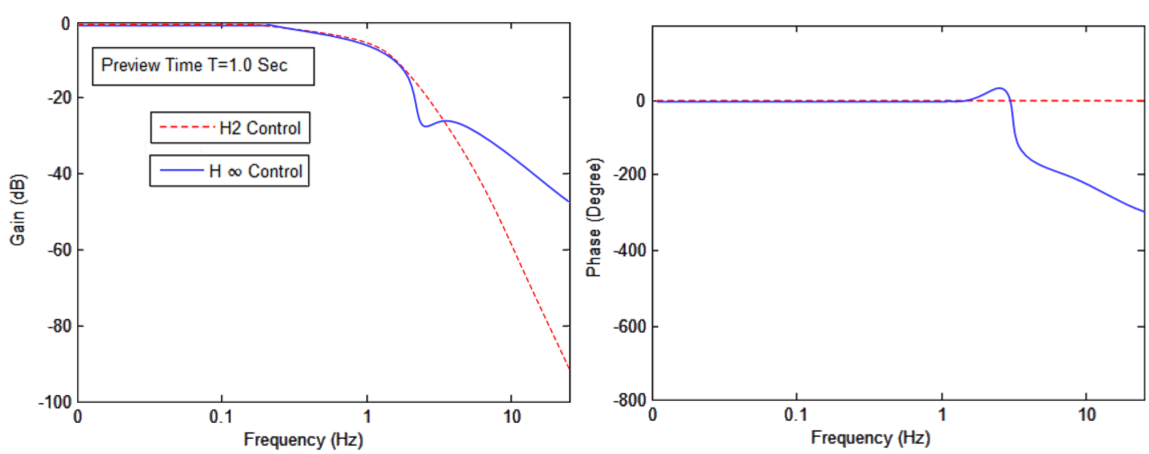

Fig. 6. Frequency response of H_2 and H_∞ preview control

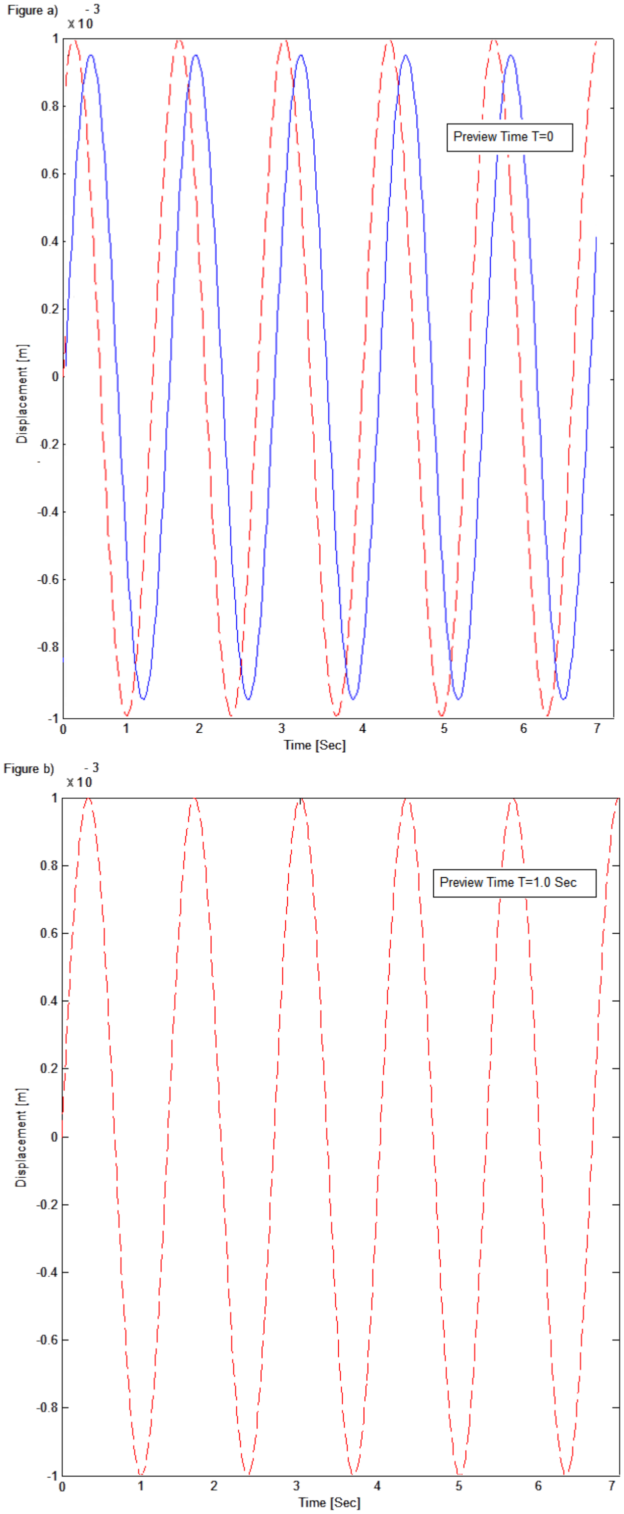

Fig. 7. Time response for sinusoidal input (0.5Hz)

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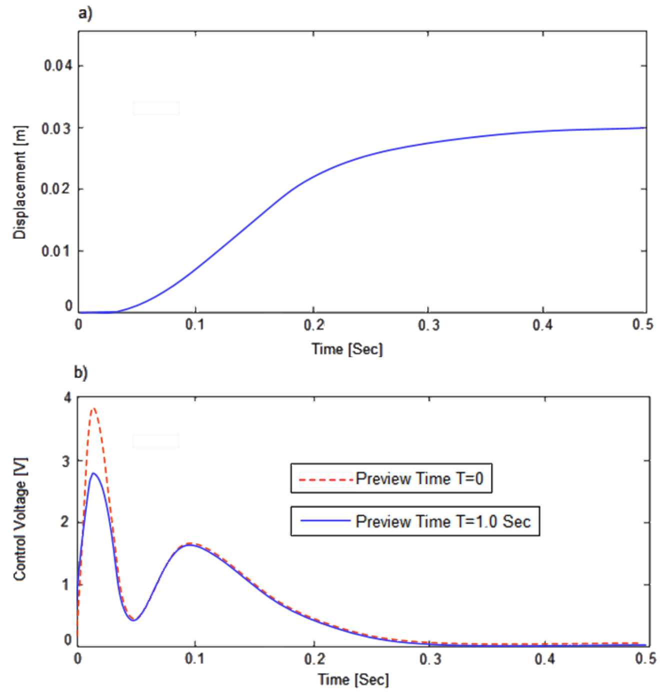

Fig. 8. Step response for H_∞ preview control

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