Fejer-Korovkin Wavelet Based MIMO Model For Multi-step-ahead Forecasting of Monthly Fishes Catches

Nibaldo Rodriguez and Lida Barba

Abstract—This paper proposes a Multiples Input-Multiples Ouput Autoregressive (MIMO-AR) model based on two stages to improve monthly anchovy catches forecasting of the coastal zone of Chile for periods from January 1958 to December 2011. In the first stage, the stationary wavelet transform (SWT) based on Fejer-Korovkin (FK) wavelet filter is used to separate the raw time series into a high frequency (HF) component and a low frequency (LF) component. In the second stage, both the HF and LF components are the inputs into a FK+MIMO-AR model to predict the original time series. The performance of the FK-MIMO-AR model is evaluated by comparing its prediction with MIMO-AR model based on SWT with Daubechies (Db) wavelet filter (Db+MIMO-AR). Results show that the FK+MIMO-AR model outperforms the Db+MIMO-AR model in terms of root mean square error, modified Nash-Sutcliffe efficiency and coefficient of determination for 15-month-ahead anchovy catches forecasting.

Index Terms—Wavelet analysis, MIMO model, Forecasting model

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1 INTRODUCTION

Citizens of fishing countries today (Chile, Peru, China, Japan, New Zealand, Mexico, etc.) are demanding that their governments develop new sustainable policies for the exploitation of fishing resources. However, the development of such policies requires an understanding of the variability of abundance of certain species in the marine ecosystem. The development of models to aid in understanding and predicting fluctuations in abundance of fishing resources is a complex task due to the dynamics underlying the marine ecosystem.

In recent years, linear regression models [1]–[3] and artificial neural network models have been proposed for 1-step-monthly time series forecasts of pelagic species [4], [5]. The disadvantage of linear regression models is the assumption that time series of pelagic species abundance are stationary. Although artificial neural networks can model the non-linear behavior of a time series, they also have some disadvantages due to the learning algorithm based on a descending gradient, as this type of algorithm shows rapid convergence to local minimums during the learning process. Gutierrez et al. [4], [5] proposed multi-layer neural network models to forecast catches in the following month (1-step-ahead) for anchoveta and sardine in northern Chile. The results obtained from the use of a neural network achieved a variance of 87%. In order to better understand the underlying dynamics of fishing resource abundance in Chile, it is necessary to develop new models to explain and predict the oscillatory behavior of pelagic resources along the Chilean coastline.

In recent decades some researchers in order to improve non-stationary time series forecasting models have used the wavelet analysis. The advantage of wavelet analysis is its ability to detect and separate high frequency and low frequency components from a non-stationary time series. After separation, each component is more regular than the original time series,

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which may help improve the forecasting performance [6], [7]. Wavelet analysis has also been evaluated successfully in $one-step-ahead$ forecast models in different areas, such as the electricity market [6], [7], the finance market [8]-[9], smoothing methods [10]-[11] and in ecological time series modeling [12], [13]. In addition, wavelet analysis at different timescales has also been used to show that climate oscillations such as the El Nino-Southern Oscillation significantly affect marine species abundance [14]-[15].

In this paper, a multi-step-ahead forecasting model of monthly anchovy catches is proposed. Our proposed forecasting model is based on two phase. In the first phase, the stationary wavelet transform (SWT) based on Fejer-Korovkin wavelet (FK) filter is used to extract a high frequency (HF) component of intra-annual periodicity and a low frequency (LF) component of inter-annual periodicity. In the second stage, both the HF and LF components are the inputs into a MIMO-AR model to predict the original time series. Besides, the proposed MIMO-AR model is compared with a MIMO-AR model based on SWT with Daubechies wavelet filter [16], [17] denoted as Db+MIMO-AR.

This paper is organized as follows. In the next section, we present hybrid multi-step-ahead forecasting model. The simulation results are presented in Section 3 followed by conclusions in Section 4.

2 PROPOSED MULTI-STEP-AHEAD FORECASTING

In order to predict the future values of time series $x(n)$, we can separate the raw time series $x(n)$ into two components by using SWT. The first extracted component x_H of the time series is characterized by fast dynamics, whereas the second component x_L is characterized by low dynamics. Therefore, in our forecasting model a time series is considered as a functional relationship of several past observations of the components x_L and x_H as follows:

$$\hat{x}(n+h) = F(z(n)) \tag{1}$$

where the h value represents forecasting horizon, m denotes lagged values of both the LF and HF components and $z(n) = [x_L(n), \ldots x_L(n-m), x_H(n), \ldots x_H(n-m)]$ denotes regressor vector. Besides, the functional relationship $F(\cdot)$ in this paper is estimated by using a MIMO-AR model. The proposed MIMO-AR model calibrates only one MIMO-AR model to predict the h future values. The following equation is used to represent the linear MIMO-AR model:

$$[\hat{x}(n+1), \hat{x}(n+2)\ldots, \hat{x}(n+h)] = F[z(n)] + e(n) \tag{2}$$

The MIMO-AR model is used to estimate the function $\hat{F}(\cdot)$. Given a set of training data $z_i, d_i, i = 1, \ldots, N$, with $z_i \in R^{2m}$ and $d_i \in R^h$, then the output forecasting in matrix form is obtained as

$$Y = ZA \tag{3}$$

where Y is the matrix dependent variables of N rows by h columns, Z is the regressor matrix of N rows by $2m$ columns and A is the parameters matrix of $2m$ rows by h columns. In order to estimate the parameters A the linear least squares method is used, which is given as

$$A = Z^{\dagger}Y \tag{4}$$

where $(\cdot)^{\dagger}$ denotes the Moore-Penrose pseudoinverse [18].

2.1 Stationary wavelet transform

Let $x(n)$ denote the value of a time series at time n, then $x(n)$ can be represented at multiple resolutions by decomposing the signal on a family of wavelets and scaling functions [10]. The approximation (scaled) signals are computed by projecting the original signal on a set of orthogonal scaling functions of the form:

$$\phi_{jk}(t) = \sqrt{2^{-j}}\phi(2^{-j}t - k) \tag{5}$$

or equivalently by filtering the signal using a low pass filter of length r, $h = [h_1, h_2, ..., h_r]$, derived from the scaling functions. On the other hand, the detail signals are computed by projecting the signal on a set of wavelet basis functions of the form

$$\psi_{jk}(t) = \sqrt{2^{-j}}\psi(2^{-j}t - k) \tag{6}$$

or equivalently by filtering the signal using a high pass filter of length r, $g = [g_1, g_2, ..., g_r]$, derived from the wavelet basis functions. Finally, repeating the decomposing process on any scale J, the original signal can be represented as the sum of all detail coefficients and the last approximation coefficient.

In time series analysis, discrete wavelet transform (DWT) often suffers from a lack of translation invariance. This problem can be tackled by means of the un-decimated stationary wavelet transform (SWT). The SWT is similar to the DWT in that the high-pass and low-pass filters are applied to the input signal at each level, but the output signal is never decimated. Instead, the filters are up-sampled at each level.

Consider the following discrete signal $x(n)$ of length N where $N = 2^J$ for some integer J. At the first level of SWT, the input signal $x(n)$ is convolved with the $h_1(n)$ filter to obtain the approximation coefficients $a_1(n)$ and with the $g_1(n)$ filter to obtain the detail coefficients $d_1(n)$, so that:

$$a_1(n) = \sum_k h_1(n-k)x(k) \tag{7a}$$

$$d_1(n) = \sum_k g_1(n-k)x(k) \tag{7b}$$

because no sub-sampling is performed, $a_1(n)$ and $d_1(n)$ are of length N instead of $N/2$ as in the DWT case. At the next level of the SWT, $a_1(n)$ is split into two parts by using the same scheme, but with modified filters h_2 and g_2 obtained by dyadically up-sampling h_1 and g_1.

The general process of the SWT is continued recursively for $j = 1, ..., J$ and is given as:

$$a_{j+1}(n) = \sum_k h_{j+1}(n-k)a_j(k) \tag{8a}$$

$$d_{j+1}(n) = \sum_k g_{j+1}(n-k)a_j(k) \tag{8b}$$

where h_{j+1} and g_{j+1} are obtained by the up-sampling operator inserts a zero between every adjacent pair of elements of h_j and g_j; respectively.

Therefore, the output of the SWT is then the approximation coefficients a_J and the detail coefficients $d_1, d_2, ..., d_J$, whereas the original signal $x(n)$ is represented as a superposition of the form:

$$x(n) = a_J(n) + \sum_{j=1}^{J} d_j(n) \tag{9}$$

The wavelet decomposition method is fully defined by the choice of a pair of low and high pass filters and the number of decomposition steps J.

2.2 Measures of accuracy applied in the model performance

In this study, three criteria of forecasting accuracy called root mean square error (RMSE), modified Nash-Sutcliffe efficiency (mNSE) and coefficient of determination (R2) were used to evaluate the forecasting capabilities of the proposed forecasting models, which are defined as

$$RMSE = \sqrt{\frac{1}{L} \sum_{i=1}^{L} \left(x(i) - \hat{x}(i)\right)^2} \tag{10}$$

$$mNSE = 1 - \frac{\sum_{i=1}^{L} \left|x(i) - \hat{x}(i)\right|}{\sum_{i=1}^{L} \left|x(i) - \bar{x}\right|} \tag{11}$$

$$R2 = 1 - \frac{\sum_{i=1}^{L} \left(x(i) - \hat{x}(i)\right)^2}{\sum_{i=1}^{L} \left(x(i) - \bar{x}\right)^2} \tag{12}$$

where $x(i)$ is the actual value at time i, $\hat{x}(i)$ is the forecasted value at time i, $\bar{x}$ is the mean of observed data and L is the number of forecasts.

3 EXPERIMENTS AND RESULTS

In this section, we apply the proposed wavelet MIMO-AR model for multi-step-ahead forecasting. The data set used corresponded to landing of anchovy in the south of Chile. These samples were collected monthly from 1 January 1958 to 31 December 2011 by the National Fishery Service of Chile (www.sernapesca.cl). The raw anchovy data set have been normalized to the range from 0 to 1 by simply dividing the real value by the maximum of the appropriate set. On the other hand, the original data set was also divided into two subsets. In the first subset the 80% of the time series were chosen for
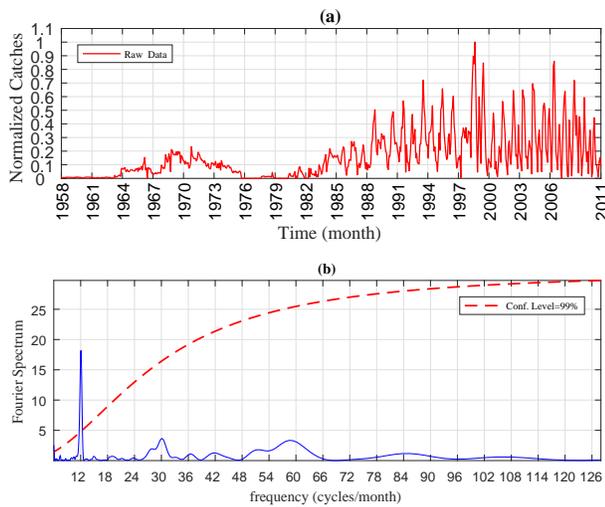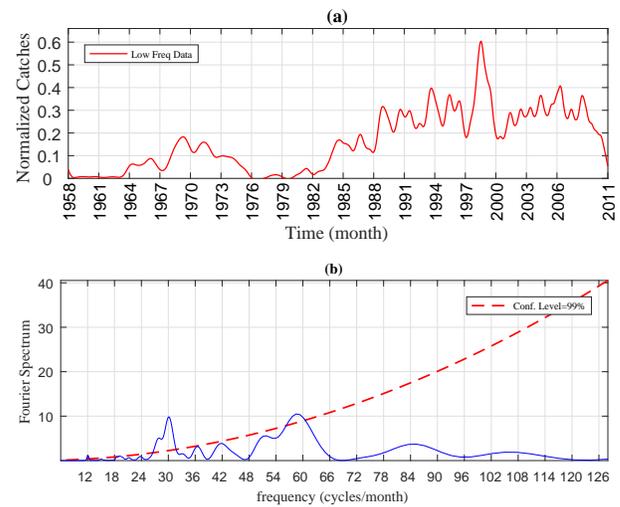
Fig. 1. Monthly anchovy catches

Fig. 2. Low frequency monthly anchovy catches

Fig. 3. High frequency monthly anchovy catches

the calibration phase (parameters estimation), whereas the remaining data set were used for the testing phase.

The normalized raw time series and the Fourier power spectrum are present in the Figure 1(a) and 1(b); respectively. The red thick line in Figure 1(b) designates the confidence level against red noise spectrum. From Figure 1(b) it can be observed that there are one peak of significant power, whose peak has periodicity of 12 months. After we applied the Fourier power spectrum to the raw time series, we decided to use 3-level SWT due to the significative peak of 12 months. Both the HF and LF times series are presented in Figures 2 and 3; respectively, whereas the power spectrum of both time series are illustrated in Figure 2(b) and 3(b); respectively. Find the order of the MIMO-AR model is a complex task, but here we will use 30 months due to significant period of the low frequency component.

The multi-step-ahead forecasting methodology used in this paper is based on SWT combined with MIMO-AR model and in order to evaluate the contribution of modeling the monthly anchovy catches using FK+MIMO-AR model, the latter is compared to Db+MIMO-AR model. The SWT is implemented by evaluating a wavelet families: (i) FK4, (ii) FK6, (iii) Daubechies db2 (iv) Daubechies db3. The results of the forecasting performance of different wavelets are reported in Figure 4. From Figure
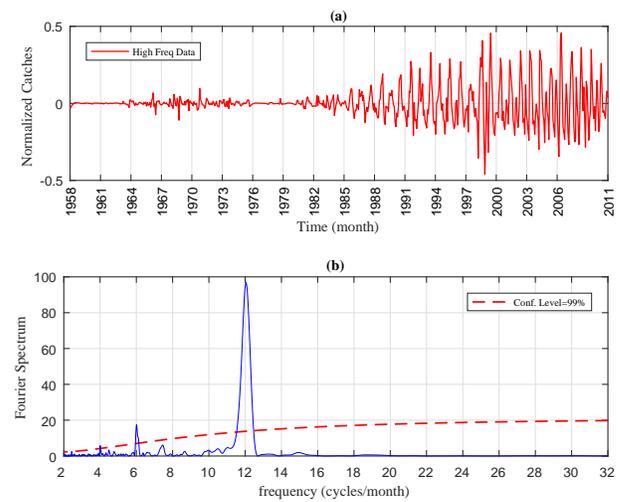
4 it is observed that the accuracy decreases as the time horizon increases; therefore the best accuracy was obtained for the nearest months, and the lowest accuracy was obtained for the farthest months. Also, from Figure 4 it is seen that the FK4-wavelet (also Db2-wavelet) seems to perform better than other wavelets due to their good localization ability. The MIMO-AR model using FK4-wavelet has a mNSE equal to 90.45% whereas the models based on db2-wavelet and Db3-wavelet yielded results with low mNSE values equal to 80.98% and 67.46%, respectively. The Figures 5 and 6 show the results obtained with the MIMO-AR(30) for 15-month-ahead anchovy catches forecasting during the testing phase. Figure 5(a) provides
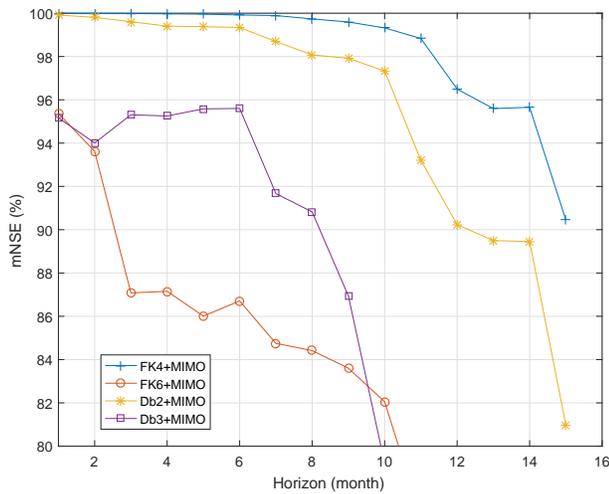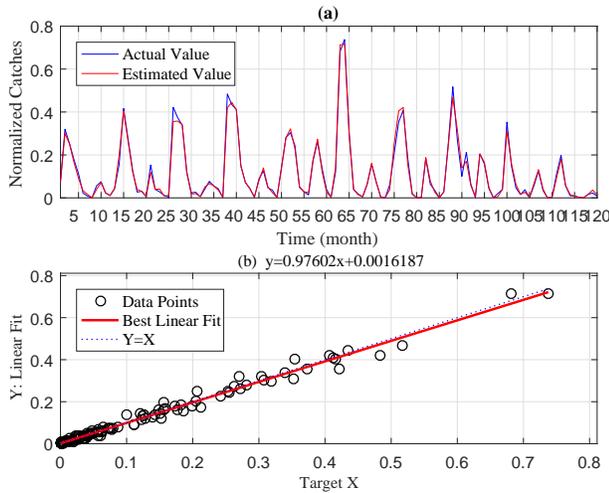
Fig. 4. FK+MIMO-AR versus Db+MIMO-AR

Fig. 5. FK4+MIMO-AR: Fifteen-month-ahead forecasting for test data set

Fig. 6. Db2+MIMO-AR: Fifteen-month-ahead forecasting for test data set

data on observed monthly anchovy catches versus forecasted catches;this forecasting behavior is very accurate for testing data with a RMSE 0.017 and a mNSE of 90.45%. On the other hand, from Figure 5(b) it can be observed a good fit to a linear curve with a coefficient of determination of 98.63%.

Figures 6(a) and 6(b) show the results obtained with the Db2+MIMO-AR(30) forecasting model during the testing phase. Figure 6(a) illustrates the observed data set versus forecasted data set, which obtains a RMSE and a mNSE of 0.034 and 81%; respectively. On the other hand, Figure 6(b) shows the scatter curve between observed values and forecasted values with a R2 of 94.75%.
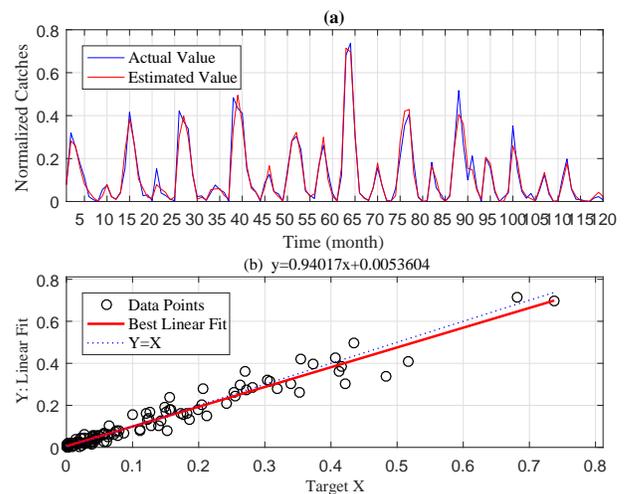
4 CONCLUSIONS

In this paper was proposed a multi-step-ahead forecasting model to improve prediction accuracy based on stationary wavelet decomposition combined with MIMO-AR model. The reason of the improvement in forecasting accuracy was due to use Fejer-Korovkin wavelet filter to separate both the LF and HF components of the raw time series, since the behavior of each component is more smoothing than raw data set. It was show that the proposed FK4+MIMO-AR model achieves a mNSE of 90.45% and a R2 of 98% for 15-month-ahead anchovy catches forecasting. Besides, the experimental results demonstrated a better performance of the proposed model when compared with a Db2+MIMO-AR prediction model. Finally, hybrid forecasting model can be suitable as a very promising methodology to any other pelagic species.

ACKNOWLEDGMENTS

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