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An Analysis on the Corrective Formula for Multiple Choice Questions

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Abstract. In multiple choice questions, an element inherent to this type of evaluation arises that, due to the same evaluation format, that is by chance, a percentage of the correct answers do not reflect the mastery of the question content being evaluated. For this reason, in many cases, a correction formula is applied that penalizes incorrect answers, leaving unanswered questions without penalty. Of course, it is necessary to mention this to the applicant prior to the start of the test. The penalty in the case of questions with k alternatives where only one is correct and all the others are incorrect, depends on the number of options and in most applications consists of subtracting a value of $1/(k-1)$ for each incorrect answer. This formula assumes that the student knows the correct answer, or that they ignore the question and answer randomly. This is difficult to achieve in practice, since the student usually has partial knowledge of the question theme, helping them to eliminate one or more alternatives. In this paper, an analysis is made of the value of the penalty in cases where partial knowledge on the part of the applicant is considered, which allows them to discard one or more distracters.

Keywords. Partial knowledge, penalty formula, distracters, testb, multiple choice.

1 Introduction

In a multiple-choice exam, a student may frequently answer a question correctly by chance or if they have partial knowledge and can eliminate some incorrect answers and then choose correctly at random. Intuitively we can see that the greater the number of possible answers, the more difficult it is to choose correctly at random, however, for a greater number of answers, the difficulty of finding a third or fourth distracter becomes extremely difficult, as it is not easy for the examiner to write questions with four or five equally plausible answers.

This can be seen in many questions where there are distracters that nobody or almost nobody chooses, regardless of how much the student knows [5, 8, 22, 23].

By reducing the number of alternatives, the least acceptable ones are eliminated, building more plausible and homogeneous alternatives in content, thereby reducing the clues that make it easier to respond correctly with only partial knowledge of the problem.

Instead, increasing the number of questions to cover more content and increase reliability may be more beneficial [20, 9, 24]. Most texts routinely recommend using four or five answers, although four is possibly the most frequent number, despite the difficulty of constructing this type of test.

The probability of choosing correctly at random is higher when there are only three options, but frequently when there are more than three possible answers, many of them are not admissible.

Many researchers tend to favor the three alternative approach, since four questions with three alternatives seem preferable to three questions with four alternatives [14, 17, 19, 21].

There are several studies that confirm that three answers are sufficient and perhaps better due to the quality of the alternatives, thus reducing flaws in homogeneity and plausibility, since a greater number of alternatives increases the risk of constructing irrelevant distracters and at the same time increasing the heterogeneity of the content.

The analysis of the distracters is carried out by means of indicators that compare the skill level of the individual with the choice of alternatives [12, 1, 16]. It is clear that any method of educational evaluation must provide the most accurate measurement possible.

In a multiple-choice exam, an intrinsic element of this evaluation format is the possibility of answering at random. Given that in this type of question, there are often deficiencies in its construction or that the respondent only has partial knowledge, it can be inferred that the respondent can deduce the correct answer or discard one or more options to later choose at random, even with a low level of knowledge.

This means that chance is a source of measurement error, so it should be taken into account that a percentage of the evaluation does not reflect the progress and results of their learning. In order to obtain a rating that is less affected by chance, in most cases a correction formula is applied.

In an attempt to dissuade students from guessing their responses, correction formulas have been implemented that penalize questions answered incorrectly. The most commonly used in questions where there are k possible alternatives of which only one is correct, is that which penalizes each wrongly answered question with:

$$m = \frac{1}{(k-1)}. \quad (1)$$

In the event that the student chooses at random between three alternatives of which only one is correct, each wrongly answered question is penalized by 0.5.

The problem with this formula is that the following two assumptions are implicitly made, with the second being hard to fulfill:

– The student knows the answer and answers correctly.

– The student does not know and randomly chooses one of the alternatives (where all possible answers are equally attractive to the student).

In other words, the assumption that justifies the application of this formula is that the student knows the correct answer with confidence, or the student totally ignores the question and answers randomly.

It is worth mentioning that there are other corrective formulas also developed to penalize wrong answers, but ultimately they are based on the assumption that wrong answers are unsuccessful attempts to get the answer right without actually knowing the correct answer, and so their purpose is to dissuade the student from responding when they are not sure of the right answer [15, 18, 25].

The choice of an option can also be made for reasons unrelated to skill, either by pure chance due to ignorance of what to answer or due to incorrect reasoning, or because the option seems attractive due to the student?s incomplete knowledge of the subject. Studies show that when the correct answer is ignored, the answers are not chosen randomly as the correcting formula assumes.

A student may at least know that some answer is incorrect and randomly choose among the rest, or perceive that one answer is more probably correct than others [2, 3, 10]. In order for the candidate to refrain from using

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chance, they are encouraged to leave the question unanswered where they have doubt, since unanswered questions are not penalized.

On the other hand, incorrectly answered questions are penalized, and accordingly it is preferable not to answer when they do not know an answer. However, in many cases this warning is ignored, especially when the student has partial knowledge and accepts the risk, and answers despite being warned [6, 13].

There are also individuals who do not guess and prefer not to answer even though wrong answers are not penalized, so choosing correctly at random does not have the same advantages for everyone.

Likewise, it is not unknown that for some objective tests, although they may be well constructed, to stimulate poor consideration in the student, due in part to the opportunity to answer using chance in case of not knowing or having partial knowledge of the problem, despite a warning that a correction formula is being used [15, 6, 25].

With the standard method of penalty calculation, if the student does not know the answer or has partial knowledge of the problem, it is likely that they will answer the question, despite knowing that a penalty will be used if they answer incorrectly. This is probably because the correction formula penalty is not the correct value or is underestimated [6].

The standard penalty formula does not take into account the partial knowledge of the student, causing the penalty value to be below its optimal value, which means that the decision to omit an item may reflect discrimination in the students that ends up benefiting those less risk averse and penalizes risk avoiders [11].

This discrimination is auxiliary to the reduction of the measurement error, which is why a greater penalty is advocated. It has been shown that women often act in a more risk averse manner, compared to men [11]. In other words, women are more likely to leave an answer blank when they are not sure, which puts them at a disadvantage compared to men.

Some authors [11, 7, 4]) establish that an effective penalty that discourages guessing exceed the standard penalty by a considerable amount, and thus increase the reliability and validity of the test. Simulations have also been carried out [11] where it is shown that the optimal penalty is relatively high.

None of these methods meet the requirements of a perfect penalty system for assessing knowledge in multiple choice tests. In order to study a more realistic penalty value, we consider the case where partial knowledge of the supporter is considered, and propose a penalty value that differs from the traditional value as mentioned in the aforementioned studies.

To motivate a deeper study in the student taking a multiple choice exam, it is necessary that the student is aware of the importance of having precise knowledge, and that imprecise knowledge is not useful in their professional training [2] and Sabers 1988. For this reason, it is necessary to develop research on the true value of the penalty so that it reduces the likelihood of the student answering without having precise knowledge.

2 Main Results

In this work, an investigation is made on the penalty value in multiple choice exam questions, taking into account the partial knowledge of the student. It can be used where they do not know the answer and respond randomly, or if they have partial knowledge that helps them rule out some options and then proceed to answer randomly. The problem that we will address consists of multiple choice questions with three alternatives, where one of them is correct and the other two incorrect.

The omitted questions are not penalized, instead the wrongly answered questions are penalized. On these assumptions, we are interested in measuring in a certain way the actual knowledge of the student. Considering the above scenarios, we define the following events shown in Table 1. The events A_1, A_2 and A_3 are disjoint

Table 1. Possible events when answering a question

B	The question is answered correctly.	p	Probability of the question being answered correctly.
A_1	The correct answer is not known and the question is answered at random.	p_1	Probability of the correct answer not being known and the question answered at random.
A_2	An option is discarded and then the question is answered at random.	p_2	Probability of an option being discarded and the question then being answered at random.
A_3	The correct answer is known.	p_3	Probability of the correct answer being known.

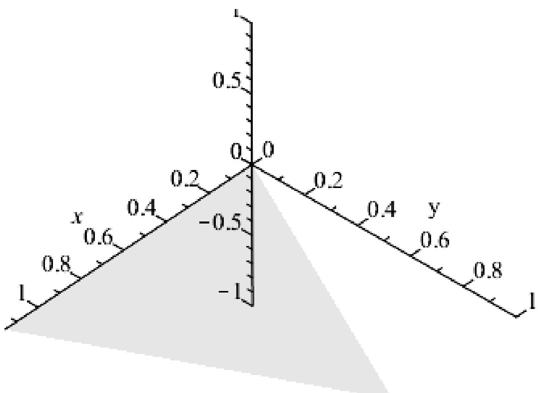

Fig. 1. Points $Q = (x,\ y)$ with coordinates $P = (p_1,\ p_2,\ p_3)$ where $p_1 + p_2 + p_3 = 1$

and the following condition holds: $p_1 + p_2 + p_3 = 1$. Applying the total probability formula, we get:

$$P(\boldsymbol{B}) = P(\boldsymbol{A_1})P\left(\frac{\boldsymbol{B}}{\boldsymbol{A_1}}\right) + P(\boldsymbol{A_2})P\left(\frac{\boldsymbol{B}}{\boldsymbol{A_2}}\right) + P(\boldsymbol{A_3})P\left(\frac{\boldsymbol{B}}{\boldsymbol{A_3}}\right), \quad (2)$$

$$P(\boldsymbol{B}) = p_1\left(\frac{1}{3}\right) + p_2\left(\frac{1}{2}\right) + p_3 \cdot 1. \quad (3)$$

We now consider the random variable X defined as:

$$X = \begin{cases} 1 & \text{if the question is answered correctly,} \\ -m & \text{if the question is answered incorrectly,} \end{cases} \quad (4)$$

where the value of m is the penalty in case of the student having answered the question incorrectly, when the following condition is met:

$$E(X) = p_3. \quad (5)$$

That is, we expect that on average the student answers correctly. Calculating the expected value we have:

$$E(X) = 1\left[p_1\left(\frac{1}{3}\right) + p_2\left(\frac{1}{2}\right) + p_3 \cdot 1\right] - m\left[1 - \left(p_1\left(\frac{1}{3}\right) + p_2\left(\frac{1}{2}\right) + p_3 \cdot 1\right)\right]. \quad (6)$$

Equating $E(X)$ to the value p_3, which is the probability of knowing the correct answer, we obtain the equality:

$$\frac{2p_1 + 3p_2}{6} + p_3 + m\left[\frac{2p_1 + 3p_2 + 6p_3 - 6}{6}\right] = p_3. \quad (7)$$

Solving the value of m we get:

$$m = -\frac{2p_1 + 3p_2}{p_1 + 3p_2 + 6p_3 - 6}. \quad (8)$$

In the previous equality m remains undefined if $p_3 = 1$, since from the condition $p_1 + p_2 + p_3 = 1$ it follows that $p_1 = p_2 = 0$ and thus the denominator would be equal to zero. However, for the value $p_3 = 1$, we can assign the value of $m = 0$, since the student answers the question correctly and would not have any penalty. The formula can be defined as follows:

$$m(p_1,\ p_2,\ p_3) = \begin{cases} -\dfrac{2p_1 + 3p_2}{2p_1 + 3p_2 + 6p_3 - 6} & \text{if} \quad p_3 \neq 1, \\ 0 & \text{if} \quad p_3 = 1. \end{cases} \quad (9)$$

Clearly, the penalty depends on the values p_1, p_2 and p_3, although we really only need two values since the third is determined by the condition $p_1 + p_2 + p_3 = 1$.

The traditional formula (1) is applied if the conditions “the student knows and answers correctly” or “the student does not know and

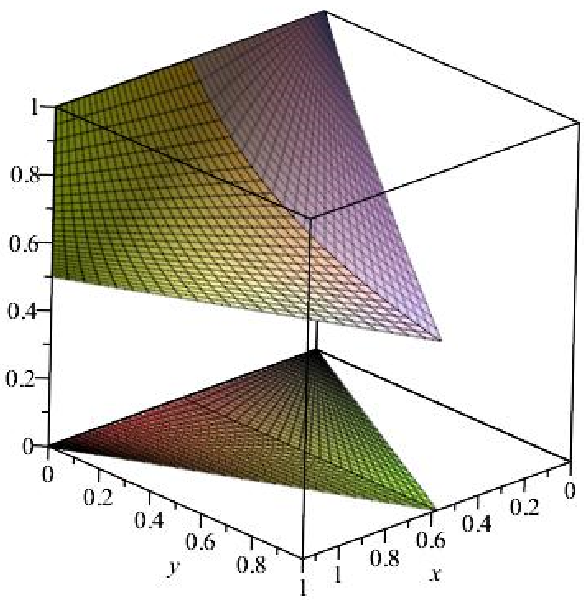

Fig. 2. Penalty function values $m(x,\ y)$

chooses one of the three alternatives at random" are met, these conditions are verified in formula (2) when $p_2 = 0$, and in such a case we have $(1 - p_3) = p_1$, and the value of the penalty is $m = 0.5$, which coincides with the value of formula (1).

We are interested in the different values that m can take for arbitrary values of $p_1,\ p_2$ and p_3 with the condition $p_1 + p_2 + p_3 = 1$.

However, finding or knowing the values of at least two of the variables $p_1,\ p_2$ and p_3, is too difficult because they depend in particular on the degree of knowledge of each student, which makes it almost impossible to try to find them in a group of students. One of the statistical techniques to solve this type of problem is to find an average value for m.

In this case the average value would be the center of mass which, once found, gives us a value of m which we can use in a general way for any values of p_1, p_2 and p_3.

Calculating the center of mass, whose value is defined by triple integrals of function (2) over the points $(p_1,\ p_2,\ p_3)$ that satisfy the condition $p_1 + p_2 + p_3 = 1$, does not seem to be a simple calculation which forces us to implement a method that reduces the dimensionality and to be able to know what the center of mass is.

For this purpose, we use a transformation that reduces the dimensionality. We only need to calculate double integrals, and that is achieved taking into account the condition $p_1 + p_2 + p_3 = 1$.

To reduce the dimensionality of the problem, we take into account that all non-negative real numbers $p_1,\ p_2$ and p_3 where $p_1 + p_2 + p_3 = 1$, can be represented inside an equilateral triangle T with sides $2/\sqrt{3}$.

Specifically, each point $Q = (x,\ y) \in T$ is identified with the point $P = (p_1,\ p_2,\ p_3)$ where each p_i is the perpendicular distance from the point $Q = (x,\ y)$ to one of the lines L_i that are part of the triangle T, $i = 1,\ 2,\ 3$, see Figure (1).

To characterize the points of the triangle T we need the equations of the lines L_2 and L_3 of T, as shown in Figure (1):

– Equation of the line L_2: $y = 2 - \sqrt{3}x$.

– Equation of the line L_3: $y = \sqrt{3}x$.

The characterization of the triangle T has the expression:

$$T = \left\{ \frac{(x,\ y)}{\begin{array}{c}\left(0 \le x \le \frac{1}{\sqrt{3}} \text{ and } 0 \le y \le \sqrt{3}x\right) \vee \\ \left(\frac{1}{\sqrt{3}} \le x \le \frac{2}{\sqrt{3}} \text{ and } 0 \le y \le 2 - \sqrt{3}x\right)\end{array}} \right\}. \quad (10)$$

If $Q = (x,\ y)$ is a point in the XY plane inside triangle T. The distance from point Q to line L_2 is:

$$p_2 = d(Q, L_2) = 1 - \frac{1}{2}\left(y + \sqrt{3}x\right). \quad (11)$$

The distance from point Q to line L_3 is:

$$p_3 = d(Q,\ L_3) = \frac{1}{2}\left(\sqrt{3}x - y\right). \quad (12)$$

Taking into account the two previous equalities, a point $Q = (x,\ y)$ inside the triangle T

corresponds to the point $P = (p_1,\ p_2,\ p_3)$ whose values are:

$$p_1 \ = \ y, \tag{13}$$

$$p_2 \ = \ 1 - \frac{1}{2}(y + \sqrt{3}x), \tag{14}$$

$$p_3 \ = \ \frac{1}{2}(\sqrt{3}x - y). \tag{15}$$

The correspondence of any point $Q = (x,\ y)$ inside the triangle T with the point $P = (p_1,\ p_2,\ p_3)$ that satisfies (3) and moreover $p_1 + p_2 + p_3 = 1$ is a one to one correspondence. Indeed, it is clear that if $Q = (x,\ y)$ is a point inside the triangle T on the XY plane, then the points p_1, p_2 and p_3 of (3) satisfy $p_1 + p_2 + p_3 = 1$.

Conversely, if we have three positive points such that $p_1 + p_2 + p_3 = 1$, then there exist numbers x, y such that the equalities (3) are fulfilled. Taking:

$$y = p_1 \quad \text{and} \quad x = \frac{1 - p_2 + p_3}{\sqrt{3}}. \tag{16}$$

Substituting the values of (3) in the formula (2) we obtain:

$$m(x,\ y) = -\frac{2y + 3\left(1 - \frac{1}{2}(y + \sqrt{3}x)\right)}{2y + 3\left(1 - \frac{1}{2}(y + \sqrt{3}x)\right) + 6\left(\frac{1}{2}(\sqrt{3}x - y)\right) - 6}. \tag{17}$$

The condition $p_3 \neq 1$ and the equalities (3) imply that $x \neq 2/\sqrt{3}$ and $y \neq 0$. Then making some simplifications we get:

$$m(x,\ y) = \frac{6 - 3\sqrt{3}x + y}{6 - 3\sqrt{3}x + 5y} \text{ if } x \neq \frac{2}{\sqrt{3}} \text{ and } y \neq 0. \tag{18}$$

Figure (2) shows the penalty function $m(x,\ y)$ defined on the XYZ plane. With the technique used we have been able to reduce the dimension of the problem which has allowed us to visualize the form of the penalty function.

Examining the shape of the graph we realize that the minimum penalty value is $m = 0.5$, which is in agreement with the statement that the traditional corrective formula (1) gives us the minimum penalty.

We can also realize that the maximum penalty is one, and that it takes this value, among other cases, when the student does not know the correct answer, answers randomly or answers wrong, and accordingly a point is subtracted.

The center of mass is defined as the point on the XY plane whose coordinates $C_{\text{mass}} = (\bar{x},\ \bar{y})$ are defined as:

$$C_{\text{mass}} = (\bar{x},\ \bar{y}) = \left(\frac{M_y}{M(x,\ y)},\ \frac{M_x}{M(x,\ y)}\right), \tag{19}$$

where:

$$M(x,\ y) = \iint_T m(x,\ y)\ dA, \tag{20}$$

$$M_x = \iint_T y\ m(x,\ y)\ dA, \tag{21}$$

$$M_y = \iint_T x\ m(x,\ y)\ dA. \tag{22}$$

For the calculation of $M(x,\ y)$, taking into account the conditions of formula (4), we obtain:

$$\begin{gathered} M(x,y) = \iint_T m(x,y)\ dA = \\ \int_0^{1/\sqrt{3}} \int_0^{\sqrt{3}x} \frac{6 - 3\sqrt{3}x + y}{6 - 3\sqrt{3}x + 5y}\ dy\ dx + \\ \lim_{\varepsilon \to 0} \int_{1/\sqrt{3}}^{(2/\sqrt{3})-\varepsilon} \int_{\varepsilon}^{2-\sqrt{3}x} \frac{6 - 3\sqrt{3}x + y}{6 - 3\sqrt{3}x + 5y}\ dy\ dx = \\ 0.22556695 + 0.19364271 = 0.41920966. \end{gathered} \tag{23}$$

For the calculation of M_x we have:

$$\begin{gathered} M(x,\ y) = \iint_T m(x,\ y)\ dA = \\ \int_0^{1/\sqrt{3}} \int_0^{\sqrt{3}x} y\ \frac{6 - 3\sqrt{3}x + y}{6 - 3\sqrt{3}x + 5y}\ dy\ dx + \\ \lim_{\varepsilon \to 0} \int_{1/\sqrt{3}}^{(2/\sqrt{3})-\varepsilon} \int_{\varepsilon}^{2-\sqrt{3}x} y\ \frac{6 - 3\sqrt{3}x + y}{6 - 3\sqrt{3}x + 5y}\ dy\ dx = \\ 0.06657315 + 0.05725798 = 0.12383113. \end{gathered} \tag{24}$$

For the calculation of M_y we have:

$$M(x,\ y) = \iint_T m(x,\ y)\, dA =$$

$$\int_0^{1/\sqrt{3}} \int_0^{\sqrt{3}x} x\, \frac{6-3\sqrt{3}x+y}{6-3\sqrt{3}x+5y}\, dy\, dx + \lim_{\varepsilon \to 0} \int_{1/\sqrt{3}}^{(2/\sqrt{3})-\varepsilon} \int_{\varepsilon}^{2-\sqrt{3}x} x\, \frac{6-3\sqrt{3}x+y}{6-3\sqrt{3}x+5y}\, dy\, dx = \tag{25}$$

$$0.08378158 + 0.14906623 = 0.23284781.$$

Using the values above, the coordinates of the center of mass $C_{\text{mass}} = (\bar{x},\ \bar{y})$ are:

$$\bar{x} = \frac{M_y}{M(x,\ y)} = \frac{0.23284781}{0.41920966} = 0.55544476, \tag{26}$$

$$\bar{y} = \frac{M_x}{M(x,\ y)} = \frac{0.12383113}{0.41920966} = 0.29539188. \tag{27}$$

Therefore:

$$C_{\text{mass}} = (\bar{x},\ \bar{y}) = (0.55544476,\ 0.29539188). \tag{28}$$

The average penalty is:

$$m(\bar{x},\ \bar{y}) = \frac{6-3\sqrt{3}\bar{x}+\bar{y}}{6-3\sqrt{3}\bar{x}+5\bar{y}} = 0.74262183. \tag{29}$$

Evidently, if we translate the problem in terms of $p_1,\ p_2$ and p_3 we get the same penalty as seen below. The point $(\bar{x},\ \bar{y})$, in terms of $p_1,\ p_2$ and p_3 see (4), corresponds to the point:

$$\left(\frac{1-p_2+p_3}{\sqrt{3}},\ p_1\right) = (x = 0.48102927,\ y = 0.29539188). \tag{30}$$

This in turn corresponds to the values:

$$p_1 = y = 0.29539188, \tag{31}$$

$$p_2 = 1 - \frac{1}{2}(y + \sqrt{3}x) = 0.37127479, \tag{32}$$

$$p_3 = \frac{1}{2}(\sqrt{3}x - y) = 0.33333333. \tag{33}$$

Substituting these values into formula (2) we obtain:

$$m = -\frac{2p_1 + 3p_2}{2p_1 + 3p_2 + 6p_3 - 6} = 0.74262183. \tag{34}$$

Which effectively gives us the same value as before. Figure (2) shows that the penalty applied using the traditional formula (1) is the minimum penalty value $m = 0.5$, and this fact can influence the student's decision to think that they gain more by answering without fully knowing the answer than by leaving the question unanswered.

However, the average penalty generated by formula (2) is much higher and can persuade the student not to answer if they do not have precise knowledge of the question and can encourage the student to study more efficiently in this type of test.

There are studies where the conclusion is reached that the penalty in the traditional corrective formula (1) is small and that it should be greater [6]. In this study we confirm two facts that have been perceived in previous studies, one is that the traditional penalty is effectively the minimum and secondly, that this penalty should be greater taking into account the partial knowledge of the student.

3 Conclusions

The penalty found in this investigation, which takes into account a partial knowledge of the student, validates in a certain way the investigations of authors who affirm that a method to discourage guessing in multiple choice tests, and thereby increasing their reliability and validity, is to set the penalty for incorrect responses high, relative to the traditional one.

Authenticity of answers and validity of scores are hallmarks of a well-constructed objective test. The importance of using a good corrective formula to minimize the guessing problem involves ensuring the validity of the inferences about the learning and the objective result obtained by the student.

The approach of the problem shown in this work, for objective multiple-choice tests, confirms in a certain way that the traditional penalty is underestimated and that a higher one would help

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dissuade students from taking the risk of answering without having precise knowledge, which would that benefits students who avoid risk.

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