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A Logical Deductive Approach to Analyze and Synthesize a Class of Linguistic Expressions Using Rabbi Moshe Chaim Luzzatto 21 Logical Names

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Abstract. This manuscript discusses soundness and completeness issues for a class of linguistic expressions using Rabbi Moshe Chaim Luzzatto 21 logical names. The presentation is divided into three. The first part deals with the conceptualization of reality and how each concept defines a name. The second part deals with how 21 logical names are used to build valid linguistic expressions. The third part deals with logical deductions and its applications. Soundness and completeness issues of the approach are addressed. The presentation finishes with an appendix where some interesting examples are provided.

Keywords. Concepts, names, logical names, expressions, syllogisms.

1 Introduction

This manuscript presents a formal and modern review of the book Sefer Hahigayon (the book of logic [5]) written by Rabbi Moshe Chaim Luzzatto, and how his methodology based on 21 logical names can not only be used to analyse and synthesize texts in Jewish literature, as the Torah and the Talmud, but it can be extended to a class of linguistic expressions.

It is shown that every linguistic expression that belongs to this class has the property of being sound and complete. Rabbi Moshe Chaim Luzzatto (1707-1746) known for his Hebrew acronym as the RAMCHAL, was a prominent Italian Jewish rabbi, kabbalist, and philosopher with a vast knowledge in religious lore, the arts, and science [4].

The presentation is divided into three parts. The first part deals with the conceptualization of reality and how each concept can be differentiated one from another by means of three characteristics: physical/mental perception, substance/attribute, and hierarchy location.

As a consequence every concept defines a point in a three dimensional space. Once a better understanding of the notion of concept has been accomplished, we can identify each concept by associating to each concept a set of words, called names.

The second part deals with how 21 logical names are used to build valid linguistic expressions. A detailed classification analysis of names in terms of its denotation or its application is provided. The 21 logical names are in detail explained.

The third part deals with a formal introduction to valid language expressions, logical inferences, and deductions, i.e., syllogisms. Soundness and completeness issues of the approach are addressed.

It is shown how using the 21 logical names it is always possible to give a procedure to compute the middle logical name shared by the two premises, and not present in the conclusion of the syllogism that is used to prove completeness.

Even more, in the case of dealing with a Kal-Vahomer syllogism the procedure is able to compute the R factor. It is important to underline that RAMCHAL [5] never gives a formal proof that the syllogism exists.

He mentions that given "any" language expression (the conclusion), there is always one syllogism, i.e., two premises (which are language expressions) that prove it, because "every" language expression is built using the 21 logical names.

This claim although is true needs a formal proof, what is done in this manuscript. It is also relevant to comment that in the process of searching for the middle logical name, he never explicitly takes care of matching the middle logical name with the two premises (see [6]), and in that aspect, differs from the computational procedure proposed here.

When RAMCHAL uses the Kal-Vahomer syllogism, he does not discuss how to compute the R factor [5]. RAMCHAL′s logic methodology, an Aristotle type of logic [7], has shown to be very powerful, as can be verified by the large number of examples to which it has been applied [6].

The reader is referred to [1], in order to compare it to the Aristotle type of logic proposed by RAMCHAL, and draw his own conclusions. The presentation finishes with an appendix where some interesting examples are given.

2 Preliminaries

The entire labor of the intellect is to try, as hardly as possible, to understand the reality in its true nature. However, the intellect may err and can arrive at a false understanding of this reality.

That is why, a formal methodology that will let the intellect differentiate between true and false and finally arrive to its true meaning is needed. This methodology is what we will call deductive reasoning or logic.

This reality, that surrounds the human being and to which he is also a part of it, is conceptualized, meaning that to each element that compounds this reality we associate the notion of a concept, and can be understood by analysing how each concept can be differentiated one from another by means of three characteristics as is next described.

The first characteristic depends on how this concept is perceived and is divided into two: the first one by the physical senses (sight, hearing, taste, etc.) and the second one by the intellect.

Some concept examples that are perceived by the physical senses are: the tree which we see, sweetness which we taste, while some that come from the intellect are: wisdom, strength, which the physical senses can not perceived but come from a mental abstraction.

The intellect perception is also divided into two: the first one called isolation, while the second one imaginative. The isolation perception is some property that does not come from the physical senses is bound up with a physical thing, and is taken by the intellect and isolates it.

As for example, the concept of color which things have but it comes from a mental process. The imaginative concept which does not come from the physical senses, is not bound up with a physical thing, but the intellect imagines (creates) it, and is also divided into two: abstraction and fantasy.

The abstraction perception is a result of an induction process of a particular physical property into a general one. As for example: animals are alive, fishes are alive, human beings are alive, so we create the concept of living being.

The fantasy perception is not a result of an induction process it merely comes from our pure imagination and it does not come from any physical reality. As an example, any mythological creature.

The second characteristic depends on if the concept is either a substance or an attribute. A substance is the main thing that is intrinsic to the concept, and not to another one, while an attribute is a feature found in the substance, it depends on it, and can not stand alone without it.

As examples we have: the human being and its wisdom, a stone and its hardness etc.

Property 1 *(Substance Properties). The attribute is attached to the substance. The attribute existence depends on the substance (no substance no attribute). An attribute can not be transferred from one substance to another one.*

We have defined what a substance is, now we will consider nine different types of attributes which are: quantity, quality, action, consequence, relation, time. position, state, and place.

– Quantity. It is a measurement in relation to the substance (how much?, or how many?) and can be continuous or discrete. As for example: 25 pounds of sugar or three eggs.

– Quality. It is the capacity that a substance has to satisfy some parameter, and it is divided into four: ability, natural disposition, constitution/formulation, and physical form. Ability: it is the quality that just the substance called human being has to perform certain actions. It is divided into three: theoretical/abstract (natural sciences, metaphysics, and mathematical sciences), social sciences (ethics, economy, and politics), and practical professions (architecture, plumbing, etc.). Natural Disposition: it is found inside the substance naturally. As for example: sharpness of the mind, strength and health of the body, etc. Constitution/Manifestation: it is how the substance is constituted and/or formulates. As for example: heavy, light, cold , hot, etc. Physical Form: the form of the substance as it is pictured and seen by the mind. As for example: the form of a man, the form of a table, etc.

Property 2 *(Quality Properties). We have more or less quality. There are similar and contrary qualities.*

– Action. It is the manifestation or effect of the substance. As for example, the heat that comes from fire.

Property 3 *(Action Properties). Every controlled action has a target. In any action we have more or less. There are opposites.*

– Consequence. It is the result of the action.

Property 4 *(Consequence Properties). In any consequence we have more or less. There are opposites.*

– Relation. It is a connection between two or more substances. As for example, a father with his son.

Property 5 *(Relation Properties). The attribute that establishes the relation is common for all the substances that share the relation.*

– Time. A temporal attribute of the substance. As for example, the measure of how far the attribute is from its beginning or its end.

– Position. The way the substance is situated in the world. As for example: standing, seating, etc.

– State. It is an attribute indicative of several acquired traits in the substance. As for example: the way he dresses, the humility, generosity, etc.

– Place. The part of the world that it occupies.

The third characteristic depends on how the different concepts can be organized or placed, according to the level each concept occupies in a defined hierarchy. We will consider two types of categorizations. The first one is called particular.

As for example: a man as part of the men, a bird as part of the birds. The second one, called generalities is divided into two: existential and general. The existential categorization is defined in terms of the notions of kind and species.

A kind is a group of entities that have at least one common characteristic upon which they may be grouped together; while the group of entities that define the kind are called species. As for example, the kind of living things which is defined in terms of the species that have the property of being alive e.g., the birds, human beings, etc.

Notice that human beings is a also a kind since it can defined in terms of men and women. Therefore, we can differentiate and define different types of kinds according to the position or level they take in the whole hierarchy. As for example: the root kind, which does not have another kind above it, or the intermediate kind, which has at least one kind above and bellow it.

Property 6 *(Kind Properties) . Any property of the kind becomes a property of its species. If the kind is denied all its species are also denied. If a kind is presupposed then there must exist species that form part of it.*

Property 7 *(Species Properties). The species depends on the kind. If a fixed species is denied not necessarily the kind is denied. If a species is presupposed then there must exist a kind that generates it.*

The general categorization which underlines the distinguishing factors is divided into three: difference, peculiar and incidental. Difference is defined in terms of an essential property which allows to differentiate between two kinds or species. As for example: an animal and a human being are both living things however, the essential property which allows to make the difference is the language.

Peculiar, is defined in terms of a non-essential property. As for example, the property of laughing that distinguishes the human being from an animal but is not essential. And finally, the incidental which, as its name says, is build in terms of a mere incident that defines the group. As for example, the people seated in the first row of a concert hall, etc.

These three characteristics associated to the notion of concept, allows us to locate and get a better understanding of every concept as a point in the three dimensional space defined by: $Physical/Mental \times Substance/Attribute \times Hierarchy$.

Once a better understanding of the notion of concept has been achieved, a mapping defined as $f : \{\mathrm{C}oncepts\} \to \{Names\}$ allows us to identify each concept by associating to each concept, a set of words (called names) in a fixed language.

3 Names and 21 Logical Names

A name is a set of letters from an alphabet which identifies each concept. More precisely it is a function that maps the set of concepts into the set of names:

$$f : \{\mathrm{C}oncepts\} \to \{Names\}. \tag{1}$$

Given a finite set of names $\{n_1, n_2, .., n_m\}$ a set of punctuation marks, a set of connectives, a set of quantifiers, and some relation R, which defines a correspondence between some of them, $R(n_l, n_k)$ will be called a basic language expression.

If in addition, there are punctuation marks, connectives, quantifiers and relations, applied to the basic language expression structure, $R(n_l, n_p, ..., n_k)$ will be called a combined language expression. In the case, that the relation R comes out to be the result of applying the 21 logical names (next explained) then, $R(n_l, n_k)$ and $R(n_l, n_p, ..., n_k)$ will be called valid language expressions.

Names are divided according to its denotation or its application. With respect to its denotation there are three types of names: specific, synonyms and homonyms. A specific name refers to a single unambiguous word denoting one and only one specific meaning. As for example: Peter, Mexican, Guadalajara. A synonymous name is a set of words that share meanings with other words, amazing and marvelous are synonyms.

Homonym is a single name with different meanings. As for example: bat, an implement used to hit a ball or a nocturnal flying mammal. Homonyms are divided into four different types: generic class, frequent, uniform, and generic name. A generic class homonym name, is a class of subjects where it is equally appropriate to use each one of them individually to name it.

As for example, the species of one kind i.e, James, Peter are all man. Frequent homonym name, it is one name with two different meanings but where it is usually more frequent to use one than the other. As for example, the word hot usually denotes something that has a high temperature, but when we say that this food is hot, we mean it is spicy. A uniform homonym name, when both meanings are equally employed.

Finally, a generic name is a name which applies to many subjects but it is an allusion to one of them. As for example, King David is known to be the biblical composer even though there are many biblical composers.

Names according to their application are divided into two: common names and technical names. Common names are those used in ordinary speech while technical names are those employed in science or art.

The 21 logical names which are used to build valid logical expressions are: cause and effect, subject and attribute, whole and part, derivation and derivative, construct and its result, definition and what is defined, division and object of division, commensurate, nonequivalent and opposite, verification and proclamation, kind and species. The last logical name, kind and species, has already been discussed, so we will proceed to explain the rest of them in detail.

3.1 Cause

A cause it is the thing that from its power an action or condition exists.

Property 8 *(Cause Properties).*

- *The cause is prior to the action or condition. To explain further, there can be two types of priority: temporal priority and logical priority (how the mind rationally perceives it). There are times when both occur at the same time however, the mind based on reason perceives one before the other. As an example sunrise and the light of the sun.*
- *Everything that exists has a cause.*
- *There is not an infinite succession of causes. (see more about this in Descartes third meditation).*
- *If we assume that there is a cause necessarily we must assume that there is an action or condition. However if the cause stops, the future action or condition ceases to exist but not its past.*

The cause can be direct, intermediate or indirect. The direct cause, is when the action follows straightforwardly from it. The intermediate cause, is when the direct cause follows straightforwardly from it. And finally, the indirect cause, is when there are many intermediate causes between it and the final action.

Remark 9 *RAMCHAL proceeds to divide the causes into four general classes: agent, goal or target, matter and form. The same number Aristotle uses as the four types of answer to the question Why?, see: [10, 2].*

The cause is divided into external and internal. The external is when the cause comes out of the action itself while the internal cause comes from the action itself. Some examples are: when the students study they learn, because man has an intellect he thinks.

3.1.1 External Cause

The external cause is divided into two, the agent and the goal. The agent is the one by which the action exists. There are two types of agent: inherent and incidental.

The inherent agent is one that produces the action by its nature or by its will. The inherent agent cause is divided into primary and secondary. The primary is when the action depends completely on him.

The primary can be just one agent or a group of agents subordinate to the primary agent. When there is a group of agents they may have the same importance in relation to the action existence or not.

An example of a primary agent acting with a group of agents where not all of them have the same importance is a presidential republic, where the president is the primary agent that governs the country, but there are a group of people, not all with the same importance, that support him. The secondary is when his contribution is not primarily responsible for the action.

As for example, studying hard is not primarily responsible for getting a degree, there are other more important requirements. The incidental agent cause is when the action is not a result of its nature or will. As an example, a runner pushing hard in a race, (but within his limits), in order to improve his record time faints.

In addition, the agent can cause the action by its proper nature or by will. As for example: fire burns and consumes by its nature while for a man singing depends on his decision.

We have explained the agent cause, we will proceed to explain the goal cause. A goal cause is when the action that it produces is a target/objective that is tried to be achieved. The goal cause can be divided into primary and secondary.

The primary goal is the agent main intention, while the secondary goal is not unintentional but is in a second level with respect to the primary goal. The goal is further divided into final goal and intermediate goal.

The final goal is when the action is expected to be achieved by means of the cause, while the intermediate acts an an intermediate step to achieve the final goal.

Property 10 *(Goal Cause Properties). The goal is what makes the cause to be executed. A cause can have many different goals. A goal cause can have positive or negative implications, i.e., it is a relative concept. The final goal is prior to the intermediate goal, at least at its mental conception, but not when it is brought into practice.*

3.1.2 Internal Cause

The internal cause is divided into two: matter and form. The matter is the one that the action is determined by the material that composes the thing. As for example, flesh and blood and other causes a human to exist.

The form is when the action is a result of an arrangement, appearance, or shape of the thing. According to the relevance that the form takes in the description of the thing, the form is divided into: essential and not-essential.

The essential is what truly makes the thing different from other things. As for example, the power of rational thinking is a result of an intellect. The not-essential is when the form describes the thing but this property is also shared with other things. As for example, a round table and a round board.

Property 11 *(Matter and Form Properties). The matter is prior to the form. The form is always more important than the matter, since it describes the true nature of the thing. The form that a thing acquires can be a result of a purely rational thought.*

3.2 Effect

The effect is the action produced by means of the cause. According to the four different causes that were introduced above, the effect is divided into: the effect of an agent which is the result it produces, the effect of a goal which is what it is achieved, the effect of the matter which is the final material obtained, and the effect of a form which is the account of what it is to be.

Property 12 *(Effect Properties). An effect can be generated by one or more causes. The effect can not have more power than its cause (second law of thermodynamics). This property allows us to deduce new properties by applying the kal-vahomer inference rule, as will be discussed later. The effect incorporates the nature of its cause. As an example, the butterfly effect where the effect obtained was already in the nature of its cause, i.e., the small change in the initial conditions.*

3.3 Subject

A subject is a thing to which various attributes are associated. The subject is divided into two: specific and nonspecific. The specific subject is when its associated attribute is intimately related to the subject, i.e., one can not exist without the other.

As for example, heat and fire. The nonspecific subject, is when the attribute is not specific to the subject. As for example, a living subject is visible however, there are no living subjects that are visible. A subject can be simple or compound. The simple is when the thing to which various attributes are associated is a single name, while the compound has two or more names.

3.4 Attribute

An attribute is a feature joined to the subject and is divided into two: intrinsic and exterior. The intrinsic attribute is attached to the subject and it can not stand alone without the subject, and it is divided into particular and non-particular. The particular is unique to a certain class of subjects, and may be either complete or non-complete.

The particular complete it is present in all the subject class at all times, as for example, feelings in the living things. The particular non-complete it is present in all the subject class but is not present at all times, as for example laughter in men is not present at all times. The non-particular can be present in subjects of a different class, as for example, movement which is present in men but it is also presented in other different classes as animals.

The exterior attribute is divided into two: concurrent and sequential. The exterior concurrent may either lie in the subject, or it is associated to the subject, or it limits the subject. Examples of each one are: clothing to the body, light to the sun, and time to a subject action. The sequential exterior, it is when it manifests before or after the subject. Examples are, spring and fall with regard to summer. When the abstract notion of attribute is used to affirm something about the subject then, we can talk about its predicate. As for example, a person who has wisdom knows what the future will bring, here wisdom is used as an attribute, while when we say he has wisdom, wisdom is used as a predicate.

The predicate is divided into essential and incidental. The essential is specific to the subject according to its existence and it is found most of the times, while the incidental, it is not specific to the subject and it is found sometimes. As examples: heat to fire and a white coat. Furthermore the predicate can be classified according to its action in: executed or potential i.e, if the action is being executed or it is in stand by. As for example, a soprano related to singing.

Property 13 *(Attribute Properties). The particular attribute can not be attributed to any other class of subjects. As for example, thinking it is a particular attribute to men and it can not be attributed to other classes of living or not living subjects. The particular complete can never be separated from the subject.*

3.5 Whole and Part

Whole means something that it is complete. As for example, a whole meal, a man. The parts are the portions of the whole.

As for example, the dessert in a whole meal, and the limbs of a man. A whole is divided into two: inherent and incidental. The inherent whole is when its parts really construct it. The incidental whole is when its parts are not essential in it at all, as for example, a subject whose parts are formed in terms of what can be said about him. The inherent is divided into two: essential and composite. The essential whole is when its parts determine its existence, i.e., its matter and form.

The composite whole is when there are parts that all together construct it. The composite whole is divided into: similar and non-similar. The similar is when its parts are the same as the whole in name and nature as for example, fire. The non-similar is when is not similar, as for example, man or house whose parts are not called man or house.

Property 14 *(Whole Properties). There is a difference between a whole and its parts and even when we consider all of them together they remain being parts. The whole is the completion of its parts. If we assume a whole, its parts are strictly assumed, but when the whole is denied, not necessarily its parts are denied.*

A part is a component from which a whole is build in conjunction with two or more components.

Property 15 *(Part Properties). Every part is not complete in relation to the whole. All what is said about the part is also said about the whole respect to this part.*

3.6 Derivation and Derivative, Construct and its Result

Derivation is the process of creating new words, called derivatives from existing words. The existing words are usually taken from causes, effects, subjects, attributes and opposites. As for example: outpatient from patient, kindness from kind, and dislike from like.

Notice that not all words come out from a derivation process. As for example, root words. The derivation process applied to a root word without changing its main meaning and effect is called a construct. As for example: photo, photograph, photocopy.

Property 16 *(Construction Properties). When any of the words in the construction is assumed, all words of the same construct process are also assumed. Any attribute that qualifies anyone of the words of the construction is inherited to the other words of the same construct process. As for example, the main purpose of a photograph is to communicate an idea, therefore a photocopy also communicates an idea.*

3.7 Definition and What is Defined, Division and Object of Division

A definition is a complete description of a fixed name called what is defined. When one is given a definition there are three relevant aspects to be considered: the parts, the requirements, and the categories.

The parts of a definition are two: kind and difference. The kind equates what is defined with other names, while the difference distinguishes it from the other names. As for example, the definition of Man is that he is a rational animal, where animal equates him with other species lacking the power of thinking, and rational makes the difference.

The requirements of a definition are two. The first is that it has to be clear and to the point. The second is that it should not say more or less of what is defined. The categories is when the definition is given in terms of what is not essential to it as: its effects, its parts. or what is associated to it.

Examples of each one are: wine as an intoxicating beverage, a man as one who has limbs, and wine as an alcoholic beverage.

Property 17 *(Definition Properties). What is affirmed or denied in the definition is inherited in what is defined. In fact they result to be logically equivalent, Definition $\Leftrightarrow$ What is Defined, i.e., one can use one or the other indifferently.*

A division describes the whole by analysing its parts. The division is divided into two: essential and nonessential, or as inherent and incidental (for their definition see 3.5).

The essential or inherent division must be such that its parts can not add up more or fall short to the whole, and that its parts must share something in common with the whole and have something else that differentiates them. As for example, Man and Animal are both living creatures however, they differ since one is rational and the other not.

Property 18 *(Division Properties). The division must be as economical as possible, i.e., minimum number of parts. The different levels that define the hierarchy must be of the same weight with respect to the division.*

3.8 Commensurate

The commensurate names are those that have in common either: that they belong to the same kind or species, or that they share some incidental attribute. It will be denoted by $A \underset{R}{\equiv} B$, meaning that A and B are equivalent under some common factor called R, or also known to be equivalent modulo R. We will consider two types of R: when R=$\{$the same amount of quality$\}$, and when R=$\{$quality, action, consequence$\}$.

Property 19 *(Commensurate Properties) . What applies to one commensurate applies to the other one, based on the quantified factor R that makes them commensurate. The commensurate equivalence holds just with respect to the R that makes them equivalent. If $C \underset{R}{\prec} A$ and $A \underset{R}{\equiv} B$ then, $C \underset{R}{\equiv} B$. Notice that it is not enough for C to be a name property that is shared by A (denoted by the relation symbol $\prec$) it has to be with respect to R. Example: judgement and kindness are equivalent modulo being both cherished, but seeking the good of others is an attribute found in kindness with respect to being cherished therefore, judgement and seeking the good of others are equivalent modulo being cherished.*

3.9 Nonequivalent Names

Nonequivalent names are those where there is at least one factor R that differentiates them. It will be denoted by $A \underset{R}{\not\equiv} B$, meaning that A and B are nonequivalent under some common factor called R (modulo R). Nonequivalent names are divided into two: different and opposites.

The different names are those where there is at least one fixed factor R, but not all of them, which makes the difference. The opposite names are those when there is a factor R that makes the difference diametrically opposed, as for example, black and white.

The different names are divided into two: commensurate different and absolute different. The commensurate different are those when R is either R={the quantity of their quality} (more or less), or R={quality, action, consequence}. The absolute different is when R={something which is completely unacceptable in one particular subject}.

Property 20 *(R={the quantity of their quality is more} Properties) (Positive Predicate) Whatever is predicated which is greater, the lesser will also be predicated. As an example, if I can lift 5kg weight, then I can lift 2kg weight. (Negative Subject) Whatever does not hold for the greater subject, no way it holds for the lesser. As an example, if he is a better long distance runner than me, and he can not run ten miles in less than one hour, surely I will not be able to run it in less than one hour.*

Property 21 *(R={the quantity of their quality is less} Properties) (Positive Subject) Whatever holds for the lesser subject, it will hold for the greater. As an example, if he got a good grade in the exam and did not know the material as well as me, of course I got a good grade. (Negative Predicate) Whatever is not predicated which is lesser, is not predicated for the greater. As an example, If I can not lift 3Kg then I can not lift 5Kg.*

This properties can be proved using the kal-vahomer inference rule which will be addressed later.

Property 22 *(R={quality, action, consequence} Properties). Whatever holds for one name will not apply to the other in that aspect that makes them different.*

Next, the opposites names are addressed. The opposites can be of five types: contradictories, privatives, kind/species, contraries, and correlatives.

We define a valuation function v from the set of names to zero and one, i.e., $v : \{\text{names}\} \rightarrow \{0, 1\}$ such that it attains the value of one, $v(a) = 1$ if a holds, and $v(a) = 0$ if a does not hold.

Contradictories opposites, are those names such that if one holds the other does not, and vice versa, and there is no possibility that both do not hold, i.e., there are no intermediate names between them. As for example, day and night. Privatives opposites, are those names which hold an attribute when there is an attribute which is absent.

As for example, sleep as the absence of insomnia. Kind/Species opposites, are those that result from the difference induced by the division made. Contraries opposites, are those names such that, if one holds the other does not and vice versa, but it is possible for both of them not to hold, i.e., there are intermediate names between them.

As for example, black and white, where red or any other color acts as an intermediate name. Correlatives opposites, are those whose relation can not be broken in the sense that in order to understand one of them the other one has to be present, and they are opposites. As for example, father opposite to son.

Property 23 *(Opposites Properties). When one of the opposites holds the other one does not. However, when one does not hold, the other one holds unless there are intermediaries names between them. Opposites according to their privatives hold only in the same subject , and if one is presupposed the other is denied. Opposites according to kind/species must be defined according to what makes them different and not in terms of what is common to them.*

3.10 Verification and Proclamation

Verification and proclamation consists on the confirmation (verification) that something is true (what is claimed). The power of what is proclaimed can not be less than the power of its verification.

4 Language Expressions

The logical language $\mathcal{L}$ of language expressions consists of: names, variables, punctuation marks, connectives, quantifiers, and R a relation symbol between names, i.e., $\mathcal{L} = \{\text{Names}, \text{Variables}, \text{Punctuation Marks}, \text{Connectives}, \text{Quantifiers}, R\}$.

Given a finite set of names $\{n_1, n_2, .., n_m\}$, a set of punctuation marks, a set of Connectives = { articles, verbs, logical connectives, etc. } a set of Quantifiers = { All, Non, This, This-Not, Some, Some-Not, All-Must, Non-Can-Be, Some-Can-Be, Some-Can-Not-Be, If-Then, If-Not-Then, If-Then-Not, If-Not-Then-Not, More, Enough,and Not-Enough }, and a relation R which defines a correspondence between two of them, its result $R(n_l, n_k)$ will be called a basic language expression.

If in addition, there are extra, punctuation marks, connectives, quantifiers and relations, applied to two or more basic language expressions, its result $R(n_l, n_p, ..., n_k)$ will be called a combined language expression.

In the case, that the relation R comes out to be the result of applying one or more of the 21 logical names, then $R(n_l, n_k)$ and $R(n_l, n_p, ..., n_k)$ will be called valid language expressions. The logical language structure is defined to be the one that interprets, names as names, and logical names as logical names, i.e., the standard language structure.

We will deal with valid language expressions that belong to $\hat{\mathcal{L}} = Cl(\mathcal{L}, R)$, the closure of $\mathcal{L}$ under $R|_{\text{21 logical names}}$, i.e., the smallest set that contains $\mathcal{L}$ and is closed under R given by the 21 logical names. When dealing with valid basic language expressions there are three aspects to be considered: the parts, the connectives, and its class.

The parts of a valid basic language expression are subject and predicate. The connectives is what associates the subject to the predicate and it can be explicit or implicit. The explicit connective is when it appears clearly, and the implicit is when not. Examples of each one of them are: the bread is not white, Peter eats. The class can be of two types, according to the quantifier being applied to the subject, or according to the manner of predication.

According to the quantifier, can be universal (All), partial (Some), and particular (This). According to the manner of predication, can be affirmative or negative. The affirmative is when the predicate applies to the subject, while the negative is when the predicate is taken off the subject. All valid basic language expressions can be simple or conditional.

The simple is when the valid basic language expression states something without any condition. The conditional is when there is a premise upon which a conclusion depends, and can be primary or secondary.

The primary conditional can be of four types: obligatory (Must), impossible (Can Not Be), possible (Can Be), and doubt (Some). Primary valid basic language expressions are called natural valid basic modal language expressions (some other modalities as temporal, logical, and so on are not included in our presentation). Examples of each one of them are: fire must burn, the parts can not be equal to the whole, he can be around people, some of the times he is sitting.

The secondary conditional can be of five types: exclusion (only, alone), exemption (except), limited (relative to), comparison (more and enough). Examples of each one of them are: he was only a teenager, there was no one else here except me, today is cold relative to yesterday, he has more money than me, and he is smart enough to overcome this challenge.

All valid basic language expressions can be true or false. The truth of any valid basic language expression can be established in two ways: either is an axiom, or there is a logical proof. Axioms can be of two types: those that are imposed by nature or are a result of the scientific method, and those that are accepted by common agreement.

Logical proofs are those procedures that achieve the truth using direct methods, i.e., logical inferences, and logical deductions or indirect methods, i.e., proofs by contradiction.

Falsehood of valid basic language expressions can be established by showing that it violates an axiom or by disproving it, e.g., by showing it implies a false valid basic language expression, and therefore the premise has to be false.

Valid combined language expressions can be of five types: logical implication, compound, disjunctive, differentiate, and comparative. Logical implication is the result of joining two valid basic language expressions A and B using if A then B.

Compound are formed by joining two or more valid basic language expressions A,B,..., C, using the connectives: furthermore, therefore, only, and, etc. Disjunctive uses the connective or. Differentiate is the one that makes the difference between A and B using even though, rather, etc. Comparative relates A and B using the connectives: as, so forth, etc.

4.1 Permutations and Oppositions

There are two methodologies which can be applied to a valid language expression in order to discover a new one: permutations and oppositions. Permutations, consists in placing the subject in the position of the predicate and vice versa without changing its true.

More precisely is an inference rule that brings a new valid language expression (what is implied) implicit in the old valid basic language expression (the premise) by permuting the subject with its predicate. This results in a new valid language expression with a different polarity (affirmative or negative), primary condition (and/or) quantification, and logical implication sign.

There are four types of permutations to consider: subject-predicate to predicate-subject, subject-predicate to predicate-non subject, subject-predicate to non predicate-subject, and subject-predicate to non predicate-non subject.

An example is, all humans are animals $\rightarrow$ $\{$some animals are humans, some animals are not non humans, no non animals are humans, all non animals are non humans$\}$. Two valid basic language expressions are opposed if one affirms what the other denies and can be of two types: contradictory or contrary.

Contradictory oppositions are when, one is true if and only if the other one is false, and there is no possibility for both to be either true or false. As an example, All humans are animals vs Some humans are not animals.

Contrary oppositions have the possibility of being both false. As for example, All humans are righteous vs No human is righteous. More about oppositions and Aristotle square of oppositions for different primary conditions and quantifiers [9].

5 $\hat{\mathcal{L}}$: Syllogism, Soundness and Completeness

Syllogism as opposed to rules of inference is a deductive reasoning methodology in which a valid basic language expression conclusion, proceeds from two given or assumed valid basic language expressions the premises, each of which shares a name with the conclusion (n_1 and n_3), and shares a common or middle name (n_2) not present in the conclusion, and therefore usually not known.

As for example, all n_1 are n_2; all n_2 are n_3, therefore all n_1 are n_3. According to the position that the middle name n_2 takes in the premises, i.e., whether it is a subject or a predicate in each, we identify four possible ways (figures) of syllogism as shown in the following table.

Figure	First	Second	Third	Fourth
Premise	$n_2 - n_3$	$n_3 - n_2$	$n_2 - n_3$	$n_3 - n_2$
Premise	$n_1 - n_2$	$n_1 - n_2$	$n_2 - n_1$	$n_2 - n_1$
Conclusion	$n_1 - n_3$	$n_1 - n_3$	$n_1 - n_3$	$n_1 - n_3$

In addition we will consider four ways in which these figures of syllogism can be qualified: quantifier syllogisms, modal syllogisms, conditional syllogisms, and kal-vahomer syllogisms ([8]).

Quantifier syllogisms are those that by using the quantifiers: All, Non, This, This-Not, Some, and Some-Not, give the extension of the subject which the predication refers to. Modal syllogisms are those whose qualifiers are: All-Must, Non-Can-Be, Some-Can-Be, and Some-Can-Not-Be.

Conditional syllogisms are those whose qualifiers are: If-Then, If-Not-Then, If-Then-Not, and If-Not-Then-Not. Kal-Vahomer syllogisms are those whose qualifiers, relative to some factor R are: More, Enough, and Not-Enough. Combining for each type of qualification those syllogisms that are sound, a vast quantity of syllogisms are obtained. A list of syllogisms, with the order given above, is next presented.

First Figure

Premise	All n_2 are n_3	All n_2 are n_3	No n_2 is n_3	No n_2 is n_3
Premise	All n_1 are n_2	Some n_1 are n_2	All n_1 are n_2	Some n_1 are n_2
Conclusion	All n_1 are n_3	Some n_1 are n_3	No n_1 is n_3	Some n_1 are not n_3

Second Figure

Premise	All n_3 are n_2	All n_3 are n_2	No n_3 is n_2	No n_3 is n_2
Premise	No n_1 are n_2	Some n_1 are not n_2	All n_1 are n_2	Some n_1 are n_2
Conclusion	No n_1 is n_3	Some n_1 are not n_3	No n_1 is n_3	Some n_1 are not n_3

Third Figure

Premise	All n_2 are n_3	No n_2 is n_3	Some n_2 are n_3	Some n_2 are not n_3
Premise	Some n_2 are n_1	Some n_2 are n_1	All n_2 are n_1	All n_2 are n_1
Conclusion	Some n_1 are n_3	Some n_1 are not n_3	Some n_1 are n_3	Some n_1 are not n_3

Fourth Figure

Premise	No n_3 is n_2
Premise	Some n_2 are n_1
Conclusion	Some n_1 are not n_3

First Figure

Premise	All n_2 are n_3	No n_2 is n_3
Premise	This n_1 is n_2	This n_1 is n_2
Conclusion	This n_1 is n_3	This n_1 is not n_3

Second Figure

Premise	All n_3 are n_2	No n_3 is n_2
Premise	This n_1 is not n_2	This n_1 is n_2
Conclusion	This n_1 is not n_3	This n_1 is not n_3

Third Figure

Premise	This n_2 are n_3	This n_2 is not n_3
Premise	This n_2 is n_1	This n_2 is n_1
Conclusion	Some n_1 are n_3	Some n_1 are not n_3

First Figure	Second Figure	Third Figure	Fourth Figure
AAA	AEE	AII	EIO
EAE	EAE	EIO	
AII	AOO	IAI	
EIO	EIO	OAO	
ARR	AGG	RRI	
ERG	ERG	GRO	

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First Figure	**Second Figure**	**Third Figure**	**Fourth Figure**
AAI	AEO	AAI	EAO
EAO	EAO	EAO	ERO
ARI	AGO	ARI	AEE
ERO	ERO	ERO	AEO
		RAI	AAI
		GAO	IAI
			RAI
AnAnAn	AnEnEn		
EnAnEn	EnAnEn		
AnInIn	AnOnOn		
EnInOn	EnInOn		
AnRnRn	AnGnGn		
EnRnGn	EnRnGn		
AnApAp	AnEpEp	AnIpIp	EnIpOp
EnApEp	EnApEp	EnIpOp	
AnIpIp	AnOpOp	InApIp	
EnIpOp	EnIpOp	OnApOp	
AnRpRp	AnGpGp	RnRpIp	
EnRpGp	EnRpGp	GnRpOp	
		ApInIp	
		EpInOp	
		IpAnIp	
		OpAnOp	
		RpRnIp	
		GpRnOp	

where:

An	All n_1 must be n_3	En	No n_1 can be n_3
Rn	This n_1 must be n_3	Gn	This n_1 can not be n_3
In	Some n_1 must be n_3	On	Some n_1 can not n_3
A	All n_1 are n_3	E	No n_1 is n_3
R	This n_1 is n_3	G	This n_1 is not n_3
I	Some n_1 are n_3	O	Some n_1 are not n_3
Ap	All n_1 can be n_3	Ep	All n_1 can not be n_3
Rp	This n_1 can be n_3	Gp	This n_1 can not be n_3
Ip	Some n_1 can be n_3	Op	Some n_1 can not be n_3

First Figure

Premise	If n_2 then n_3	If n_2 then n_3
Premise	If n_1 then n_2	If n_1 not then not n_2
Conclusion	If n_1 then n_3	If n_1 not then not n_3

Second Figure

Premise	If n_3 then n_2	If n_3 then n_2
Premise	If n_1 then not n_2	If n_1 then not n_2
Conclusion	If n_1 then not n_3	If n_1 then not n_3

Third Figure

Premise	If n_2 not then not n_3	If n_2 then n_3
Premise	If n_2 then n_1	If n_2 not then not n_1
Conclusion	If n_1 not then not n_3	If n_1 not then not n_3

Modus Ponens

Premise	If n_1 then n_3
Premise	n_1
Conclusion	n_3

Modus Tollens

Premise	If n_1 then n_3
Premise	Not n_3
Conclusion	Not n_1

Kal-Vahomer Subject

Premise	n_1 is more than n_2 (R)	n_1 is more than n_2 (R)
Premise	n_2 is enough to be n_3 (R)	n_1 is not enough to be n_3 (R)
Conclusion	n_1 is enough to be n_3 (R)	n_2 is not enough to be n_3 (R)

Kal Vahomer Predicate

Premise	more is needed to be n_1 than to be $n_3(R)$	more is needed to be n_1 than to be $n_3(R)$
Premise	n_2 is enough to be n_1 (R)	n_2 is not enough to be n_3 (R)
Conclusion	n_2 is enough to be n_3 (R)	n_2 is not enough to be n_1 (R)

Theorem 24 *$\hat{\mathcal{L}}$ is sound and complete.*

Proof. (Soundness) Given a valid basic language expression that belongs to $\hat{\mathcal{L}}$ and was proved, we have to show its truth. We proceed by induction on the length of the proof. The base case holds holds obviously since axioms are. Suppose that the hypothesis holds for all proofs of length k less than or equal to n, and take a valid basic language expression $R(n_l, n_k)$ that has been proved, i.e., there exist some valid basic language expressions (premises) that prove it, by induction hypothesis these premises are true and have as conclusion $R(n_l, n_k)$, since syllogisms are closed under truthiness, the result follows.

(Completeness) Now suppose that $R(n_1, n_3)$, which belongs to $\hat{\mathcal{L}}$ is true, therefore from its definition there is always at least one syllogism whose conclusion matches it, i.e., we have two premises $R_a(n_1, n_2) \in \hat{\mathcal{L}}$, $R_b(n_2, n_3) \in \hat{\mathcal{L}}$ with a middle logical name n_2, unknown, that proves $R(n_1, n_3)$. Since n_1, n_2 and n_2, n_3 are related by R_a, R_b there is one common logical name n_2 that is related to n_1 and n_3. In addition from the cause properties (Property 8) the conclusion $R(n_1, n_3) \in \hat{\mathcal{L}}$, an effect, has a cause why it is a valid language expression, i.e., why it is member of $\hat{\mathcal{L}}$, therefore there must exist a logical name n that is related to n_1 and n_3. Consequently by searching through all possible logical names n, we will be able to match it to the n_2 of $R_a(n_1, n_2)$ and $R_b(n_2, n_3)$, which we know of their existence, for that specific n_2. Therefore we have not just been able to prove $R(n_1, n_3)$ but we have given a method for computing the middle logical name n_2. Notice that in the case of dealing with a Kal-Vahomer syllogism, while searching for the n, in particular in the process of matching the first premise of the syllogism $R_a(n_1, n_2)$, (the one that uses the qualifier More, employing commensurate different names in terms of the quality of being R), we are able not just to compute the n but also the R factor. Now, if we have a proof of the two premises we are done. Otherwise, lets concentrate in the worst case scenario, i.e., when both of them have not been proved. Proceeding exactly in the same way as above, we would be able to find a proof of them. Iterating backwards, as much as needed, since by cause properties (Property 8), there is not an infinite succession of causes, we will finish up with some axioms, and this finally establishes our claim.

As an immediate consequence we get.

Corollary 25 *$\hat{\mathcal{L}}$ is sound if and only if $\hat{\mathcal{L}}$ is complete.*

Therefore, given a valid language expression, we can check its truthiness by proving it, i.e., we have not just an approach for synthesising new valid language expressions through syllogisms but also for analysing its truthiness.

Remark 26 *It is worth mentioning that Rabbi Moshe Chaim Luzzato provides a complete set of examples covering all possible logical names [6]. However, it is important to underline that he never gives a formal proof that the syllogism exists. He mentions that given any language expression, (the conclusion), there is always one syllogism, i.e., two premises (which are language expressions), that prove it because every language expression is built using the 21 names of logic. This last claim although is true needs a formal proof, and this is achieved in this manuscript by working in $\hat{\mathcal{L}}$. It is also relevant to comment that in the process of searching for the n_2 name, he never explicitly takes care of matching the n with the two premises. When the RAMCHAL uses the Kal-Vahomer syllogism, he does not discuss how to compute the R factor [5].*

For the sake of completeness, there is an addendum at the end of this manuscript where some applications of (25) are provided. In order to illustrate how the methodology works, we give the following example.

Example 27 *We are interested in finding a proof of the following valid language expression: All Man are mortal. Therefore we have to find a middle logical name n_2 such that the premises: All n_2 are mortal, and All Man are n_2, hold. Setting n_2 equal to living beings, we get that the cause why All Man are mortal is because Man is a species that belongs to the kind of living beings who have the attribute of being Mortal. Therefore we get the following proof:*

All living beings are mortal
All Man are living beings
All Man are mortal

Even more the following inference is also true: All Man are mortal $\Longrightarrow$ (both) Some Man are mortal, and this Man is mortal. We also get that its opposite, Some Man are not mortal, is false. We also obtain that you, me and even Socrates are mortal by an immediate application of the kind properties (6). It is also possible to give an indirect proof by setting n_2 equal to elementary particles because Man internal cause, matter, i.e., elementary particles decay, and therefore have the attribute of being transient. Therefore we get the following proof:

All elementary particles are transient
All Man are elementary particles
All Man are transient

and since, transient is commensurate to mortal under R=$\{$the quantity of their existence$\}$, by commensurate properties (19) we get that All Man are mortal. Indeed both, transient and mortal, are synonyms names for the same concept. Next, lets show that Socrates is mortal holds. Setting n_2 equal to Man, where Socrates belongs to the kind of Man whose attribute is being mortal, we get that the premises: All n_2 are mortal, and Socrates is a n_2, hold, therefore the following syllogism proves our claim:

All Man are mortal
Socrates is a Man
Socrates is mortal

6 Conclusions

This works gives me the opportunity of sharing, with a large number of scientists, some of the contributions done by one of the greatest scholars of Medieval Jewish thought, Rabbi Moshe Chaim Luzzatto. The manuscript presents a formal and modern view of how the methodology introduced by the RAMCHAL for studying texts in Jewish literature, can be easily adapted for studying linguistic expressions. Soundness and completeness issues are addressed. A procedure used in the completeness proof for computing the middle logical name and the R factor is given, and is put into practice in some application examples.

7 Addendum

In this appendix the main result of this manuscript (25) is bring into play.

Example 28 *We are interested in finding a proof of the following valid language expression: Some physical bodies can not be stones. Therefore we have to find a middle logical name n_2 such that the premises: Some physical bodies must be n_2, and No stones can be n_2, hold. Setting n_2 equal to speaking beings, we get that the cause why some physical bodies are not stones is because speaking beings and stones are different species that belong to the kind of physical bodies. Therefore we get the following proof:*

No stones can be speaking beings
Some physical bodies must be speaking beings
Some physical bodies can not be stones

Now, lets assume that that we have a proof of: Some physical bodies must be speaking beings, but not of: No stones can be speaking beings. Therefore, we have to find a proof of it, i.e., we have to find a middle logical name n_2 such that the premises: All speaking beings must be n_2, and No stones can be n_2, hold. Setting n_2 equal to men, we get that the cause why No stones can be speaking beings is because men have the attribute of speaking and men and stones are species opposites. Therefore we get the following proof:

All speaking beings must be men
No stones can be men
No stones can be speaking beings

If we have a proof of both premises, then we are done, otherwise we will have to proceed exactly as above, iterating backwards, as much as needed, until both premises have a proof, or they finish being axioms.

Example 29 *We are interested in finding a proof of the following valid language expression: Some men are Seniors. Therefore we have to find a middle logical name n_2 such that the premises: All n_2 are Seniors, and Some men are n_2, hold. Setting n_2 equal to human beings over sixty years old, we get that the cause why Some men are Seniors is because human beings over sixty years old are a part of the whole called men and by definition they are called Seniors. Therefore we get the following proof:*

All human beings over sixty years old are Seniors
Some men are human beings over sixty years old
Some men are Seniors

Example 30 *We are interested in finding a proof of the following valid language expression: The descendants of Patriarch Jacob can not be counted. Therefore we have to find a middle logical name n_2 such that the premises: No way n_2 can be counted, and The descendants of Patriarch Jacob can be likened to n_2, hold. Setting n_2 to Stars, we get that the cause why The descendants of Patriarch Jacob can not be counted is because Stars have the attribute of being infinite numerable and in the Sefer Torah is written that the descendants of Patriarch Jacob are commensurate to the number of Stars. Therefore we get the following proof:*

No way Stars can be counted
The descendants of Patriarch Jacob can be equivalent to Stars (R=$\{$number$\}$)
The descendants of Patriarch Jacob can not be counted

Example 31 *([3]) We are interested in finding a proof of the following valid language expression: Divine disapproval is enough to cause being in isolation in shame for seven days (R). Therefore we have to find a middle logical name n_2, and a nonequivalent factor R such that the premises: Divine disapproval is more than n_2 (R), and n_2 is enough to cause being in isolation in shame for seven days (R), hold. Setting n_2 to Parental disapproval, we get that the cause why Divine disapproval is enough to cause being in isolation in shame for seven days (R) is because Divine disapproval and Parental disapproval are nonequivalent (commensurate different) names in terms of the quality of R=$\{$significant$\}$, and the effect of a Parental offense is to stay in isolation in shame for seven days (significant). Therefore we get the following proof:*

Divine disapproval is more than Parental disapproval (significant)
Parental disapproval is enough to cause being in isolation in shame for seven days (significant)
Divine disapproval is enough to cause being in isolation in shame for seven days (significant)

Some other applications of Kal-Vahomer syllogisms were given in Properties 20 and 21, the details are left to the reader.

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