Normalization of a 3D-Shape Similarity Measure with Voxel Representation

Normalización de una Medida de Similitud para Formas 3D con Representación en Voxels

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Abstract
In this paper we study some properties of 3D objects such as *compactness*, the *work* done in object transformations and *the number of voxels* to be moved in order to normalize a similarity measure of an appropriate set of 3D objects. Voxel representation and scale normalization allow us to find the total distance of a set of voxels from one object to another. For these purposes, the comparison of objects is achieved by superimposing their centers of mass, using principal axes for their orientation, and Hungarian algorithm for optimal matching in bipartite graphs. All these aspects are determinant in obtaining the minimum work that needs to be done in the corresponding transformations. We present experimental results by including irregular objects taken from the human body.
Keywords: Similarity measure; compactness; transforming; positive voxels.

Resumen
En este artículo estudiamos algunas propiedades de objetos 3D tales como la *compacidad,* el *trabajo* realizado en la transformación de los objetos y el *número de voxels* a mover para normalizar una medida de similitud de un conjunto apropiado de objetos 3D. La representación con voxels y la normalización de la escala nos permite encontrar la distancia total de un conjunto de voxels de un objeto a otro. Para estos propósitos, la comparación de los objetos se logra al superponer sus centros de masa, usando ejes principales para su orientación, y el algoritmo Húngaro para apareo óptimo en gráficas bipartitas. Todos estos aspectos son determinantes para obtener el trabajo mínimo realizado en las transformaciones correspondientes. Presentamos resultados experimentales al incluir modelos de objetos tomados del cuerpo humano.
Palabras clave: Medida de similitud; compacidad; transformación; voxels positivos

1 Introduction

The list of publication about comparison, registration and recognition of 3D objects is long, but there is a synthesis book [1] where it is, also, mentioned that geometric properties such as volume and surface shapes are considered to establish their similarity measures. Recently, Lee and Park [2] have found a method based on matching an object graph of a scene with the model graph of a model in a neural network to recognize geometrical 3D objects. Some authors employ distance functions to compare objects. For instance, Malandain and Rocchisani [3] based their method in searching for the smallest distance between surfaces, which is related to a minimal potential. Similarly, a 3D object recognition from 2D images is performed in [4], [5] and [6]. It is assumed, in all these research works, that objects to be recognized independently of affine transformations, namely rotation, translation and scaling.

In 1996, Bribiesca [7] compared 3D objects through a similarity measure. This similarity measure consists of the amount of work done in the movement of voxels from the so-called *positive* set **X** to the *negative* set **Y** of voxels, and such a movement can be seen as an *object transformation* as well. Bribiesca [7] implemented a heuristic algorithm to move the voxels: the first positive voxel in set **X** is moved to the closest negative voxel in set **Y** by finding the minimum Euclidean distance. The next positive voxel to move is obtained again by considering the minimun Euclidean distance among the remaining voxels in both sets. This voxel movement is iterated until the movement of the last voxel is achieved. We call this procedure the "two closest voxels (*TCV*)" algorithm, but it does not give the optimum distance. However, we compute Hungarian algorithm, as was done in [8] to find the total smallest work to match one set to the other.

A normalized similarity measure consists on giving a percentage of similarity degree between two objects. One of the recommendations for further work in Bribiesca's paper [7] is to normalize the above-mentioned similarity measure. So, as a contribution of this paper, we propose a method to normalize this similarity measure, based on the study of compactness and the work done in transforming objects.

We assume that a *3D object* is a binary spatial representation of a three-dimensional scene, in which every voxel (*volume element*) takes the value "0" or the value "1", see [9]. We also classify as an *irregular object* one that has no symmetry axes.

In this paper we show, through some experiments, that orientation given by principal axes and position of centers of mass are crucial in performing the transformation of objects.

In Section 2 we state some important concepts used in this paper, such as positive voxels, discrete compactness for three-dimensional objects, and so on. In Section 3 we show the relation between discrete compactness, work done in transforming objects and number of positive voxels in each transformation, inferring that the most compact and the least compact objects are those most dissimilar of a set of objects to be compared according to their volumetric shapes. In Section 4 we provide a manner to normalize the similarity measure, and this method gives us a good similarity degree between two objects. In Section 5 we present some experimental results to support the idea that superimposing the centers of mass and aligning objects through their principal axes improves the work done in transforming them. Finally, the conclusions of this paper are summarized in Section 6.

2 Important Concepts

Some important concepts used in this paper are:

Discrete compactness for 3D objects. The compactness measure for 3D objects composed of voxels is defined as the ratio of contact area (total surface of the faces that voxels get in touch with) to surface area of the object. Then, given a 3D object composed of *n* voxels, and contact area A_c, the discrete compactness, normalized in the interval [0,1], is given by

$$C_D = \frac{A_c - A_{c\min}}{A_{c\max} - A_{c\min}} \quad (1)$$

where $A_{c\min} = n-1$ and $A_{c\max} = 3(n-n^{2/3})$; considering the voxel as a regular hexahedron of six polygonal faces and the area of a face as the unity [10].

Common voxels. If we consider objects O_1 and O_2, which have the same centroid and are aligned by menas of principal axes, and they preserve their volume relationship after the method of volume normalization, then the overlapping of the above-mentioned objects defines the common voxels. More precisely, let the 3D binary image of O_1 be I_{O_1} and the 3D binary image of O_2 be I_{O2} . Thus, I_C is defined by

$$I_C = I_{O_1} \cap I_{O_2}. \quad (2)$$

Clearly, I_C corresponds to the common voxels of objects O_1 and O_2.

In Figure 1(a) we show the *Heart* object, while the *Sphere* object is presented in Figure 1(b). Figure 1(a) shows I_{O_1}, Figure 1(b) I_{O2}. Figure 1(c) gives the union of objects, where the number of common voxels in this case, is 7 569.

Positive voxels. These are the voxels of object O_1 that are not common to object O_2. The positive voxels are represented by the 3D binary image $\mathcal{X}$, i.e.,

$$\mathcal{X} = I_{O_1} \setminus I_{O_2}. \tag{3}$$

Figure 1(d) illustrates all positive voxels, that is to say 2 255 for this example.

Negative voxels. The negative voxels correspond to the 3D binary image $\mathcal{Y}$ that is defined by

$$\mathcal{Y} = I_{O_2} \setminus I_{O_1}, \tag{4}$$

i.e., the 3D binary image $\mathcal{Y}$ represents the negative voxels where the positive voxels can be placed. Figure 1(e) displays the negative voxels. The number of negative voxels is the same as positive voxels because they are volume normalized.

The set of positive voxels $\mathcal{X}$ is the set of voxels of the *Heart* which are not common to the *Sphere*, while noncommon voxels belonging to the *Sphere* constitute the negative set of voxels $\mathcal{Y}$.

Finding optimum matching of positive and negative voxels. Let us think on moving all positive voxels to the places of all negative voxels. So, there is a total distance to be covered. Such a distance has to be the smallest of all possibilities. There are many manners of matching one set to another. If k is the number of voxels to be moved, then k! is the number of different manners of matching them from $\mathcal{X}$ to $\mathcal{Y}$. The 3D binary images and the distances between their voxels may be considered as a *weighted complete bipartite graph* [11] with bipartition ($\mathcal{X}$,$\mathcal{Y}$), meaning

$$\mathcal{X} = \{p_i : \; i \leq k\}, \tag{5}$$

$$\mathcal{Y} = \{n_j : \; j \leq k\} \tag{6}$$

where edge $p_i n_j$ has weight w_{ij}, i.e., each weight w_{ij} corresponds to the Euclidean distance between the voxels p_i and n_j. Thus, the *optimal assignment problem* can be defined as how to find the minimum-weight perfect matching in the weighted graph and this is named *optimal matching*. A method to find an optimal matching in a weighted complete bipartite graph is the *Kuhn-Munkres algorithm* [11]. So, the covered distances of all voxels to be moved (positive voxels) can be minimized by using the Kuhn-Munkres. This minimization produces an optimum transformation of objects.

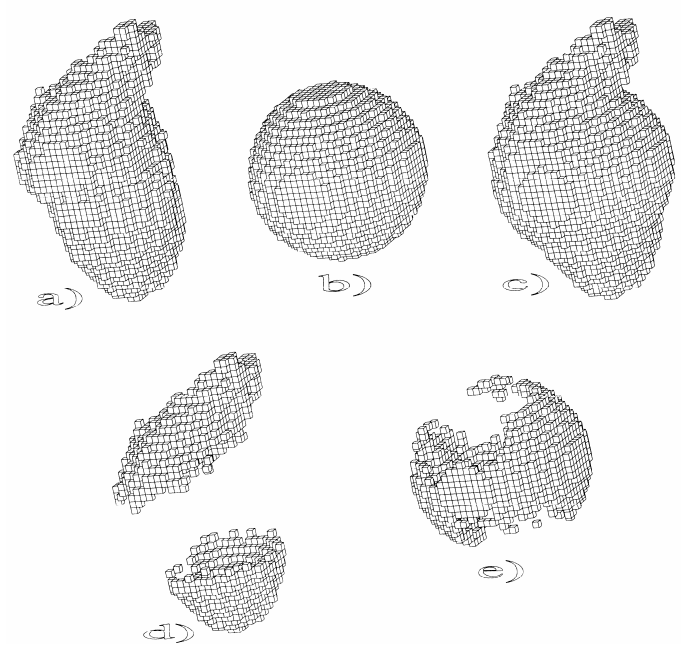

Fig. 1. Objects to be compared by "moving voxels"; a) *Heart*, b) *Sphere*, c) Superimposed objects, d) Positive voxels, e) Negative voxels

To translate every voxel from set $\mathcal{X}$ to set $\mathcal{Y}$ it is necessary to spend a specific amount of work, which represents the *work done* in transforming the source object O_1 to the designated object O_2. The concept of work for transformation of objects can be found in [7], and it is equal to the required force for moving one voxel to another and multiplied by the Euclidean distance between each pair of voxels, i.e., d($\mathcal{X}$, $\mathcal{Y}$). To establish the metric for similarity, in that paper ([7]) the author makes the force a constant and equal to one, then the work has a numerical value of a distance and it is given by the following expression,

$$W(O_1, O_2) = \sum_{i=1}^{m} d(a_i, b_j), \qquad 1 \leq j \leq m \tag{7}$$

in work units, where m is the number of positive voxels. In Eq.(7) each element from the pair (a_i, b_j) represents the location of a voxel from the positive to the negative set respectively, and it corresponds to a weight w_{ij} of a bipartite graph in an assignment problem. It also represents the distance d that each positive voxel would cover to reach its corresponding negative voxel. Furthermore we can see Eq.(7) as a similarity measure to establish the similarity degree between two volumetric shapes or objects. It can be seen that as the work increases the objects become more different in volume shape.

3 Relationship between Discrete Compactness, Work done in Transforming Objects and Number of Positive Voxels

We consider thirteen different objects shown in Figure 2, and we name this set **H**. These objects have been processed in such a manner their centroids coincide (by applying translation transforms), volume normalized (they have the same number of voxels) and they are aligned through their principal axes. As it can be seen, some of them are geometric objects (sphere, torus, etc) while the others represent organs of the human body (kidney, lung, brain, etc). All of these objects are composed of 9 824 voxels. We need to select the object having the highest discrete compactness of the set **H**, in this case, the *Sphere* that is obtained according to Eq.(1), and equals 0.9834. Accordingly, we compare the *Sphere* with the twelve remaining objects.

In Table 1 and Figure 3 we show the work done in transforming all the objects into the *Sphere*, as well as the number of positive voxels and the discrete compactness of each object. As we can see, since the smallest work in the transformation is done, the most similar object to the *Sphere* is the sphere with random noise (*Sph-noise*), followed by the sphere with a hair (*Sph-hair*), then followed by the *Brain*, etc. The most different object to the *Sphere* is the *Hand*, following *Intestine*, and so on. Note that the *Hand* has the smallest compactness, 0.7669, and although it does not have the maximum number of voxels to move, the work is relatively large (211 077.30). However, for the *Torus3*, whose compactness (0.9276) is larger than that of the *Hand* (0.7781), the work done in transforming it into the *Sphere* is smaller than the work needed with the *Hand*.

Now, by analyzing the case of the *Brain* and the *Heart*, we find that the *Brain* needs to move more positive voxels (2 385) than the *Heart* (2 255), however the work done to transform the *Brain* into the *Sphere* is smaller (24 102.13 vs. 26 104.68).

Note that once the optimum matching is known, it can be proceeded to compute the distance every positive voxel keeps to the corresponding one in the negative set.

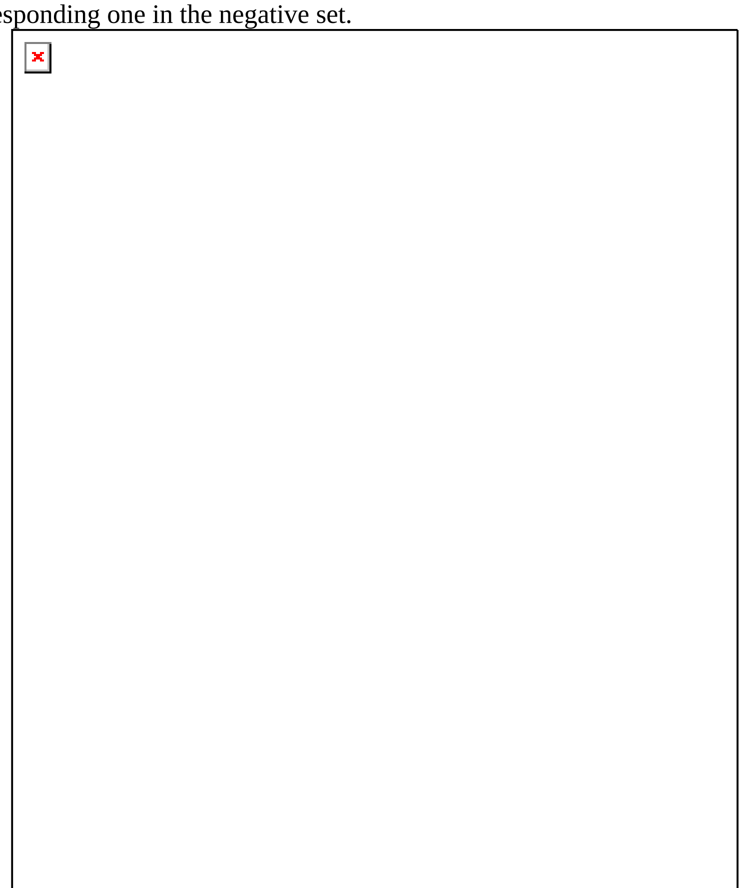

Fig. 2. The set **H** of thirteen 3D different objects, each composed of 9 824 voxels: a) *Kidney*, b) *Lung*, c)*Torus2*, d) *Sph-hair*, e) *Brain*, f) *Heart*, g) *Sphere*, h) *Torus3*, i) *Liver*, j) *Intestine*, k) *Torus1*, l) *Hand* (Skeleton), m) *Sph-noise* (sphere with random noise)

Table 1. A_c: contact area, A_s: surface area, C_D: discrete compactness, $V(+)$: positive voxels, W: work in transforming the object into the *Sphere* according to Hungarian algorithm, W_{TCV}: work according to *TCV* algorithm

Object	A_c	A_s	C_D	$V(+)$	W	W_{TCV}
Sphere	27 792	3 360	0.9834	0	0	0
Sph-noise	27 018	4 908	0.9410	419	1 292.87	1 626.29
Sph-hair	27 663	3 618	0.9763	419	6 903.98	7 217.18
Brain	27 189	4 566	0.9504	2 385	24 102.13	24 131.42
Heart	27 238	4 467	0.9531	2 255	26 104.68	26 989.93
Lung	26 967	5 009	0.9382	3 587	39 538.67	40 923.34
Torus1	27 436	4 071	0.9639	3 706	39 367.35	39 920.09
Liver	26 841	5 262	0.9313	3 817	45 420.77	48 138.05
Kidney	27 480	3 984	0.9663	4 278	57 395.35	58 395.02
Intest	24 888	9 168	0.8244	6 516	80 050.85	83 083.48
Torus2	26 197	6 549	0.8961	9 824	205 906.70	206 732.26
Hand	24 041	10 802	0.7781	8 928	211 077.30	216 229.33
Torus3	26 773	5 397	0.9276	9 820	113 735.96	114 720.34

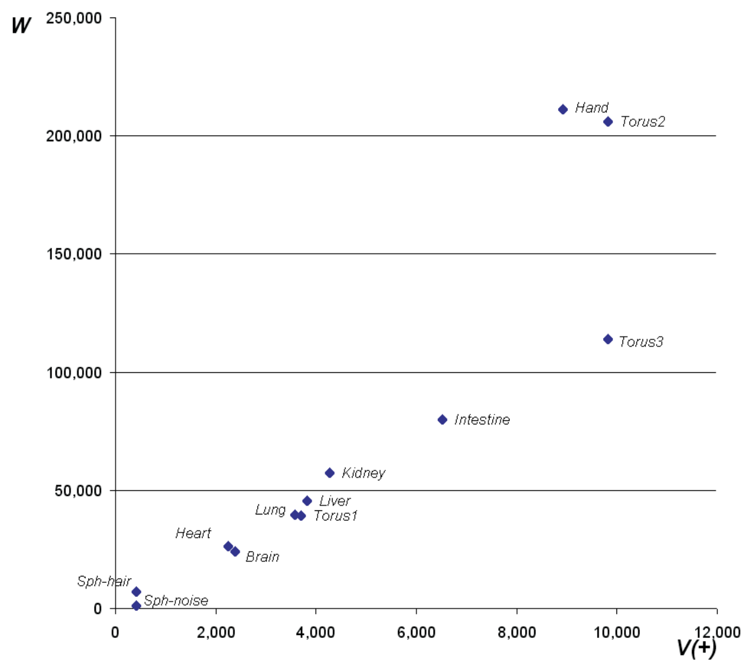

Fig. 3. Work done, in terms of positive voxels, needed to transform all objects into the *Sphere*

Another interesting situation can be observed in Figure 4, where the sphere with noise and sphere with hair have the same 419 positive voxels, even though the work done in transforming them into the *Sphere* is different. This means that transforming the sphere with a random noise is "cheaper" than transforming it into that given with a hair. The previous experiment shows that transforming the objects according to our procedure provides a robust method to estimate the similarity measure in the presence of few noise, without distort so much the shape of object. In other words, our method is more convenient than most common methods, such as simple correlation, which is very sensitive to noise. An additional advantage of our transformation is that it gives information relative to the shape of objects.

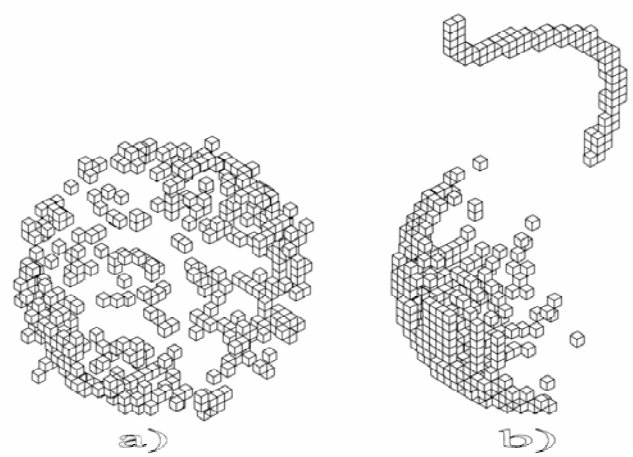

Fig. 4. a) noise distributed as random; b) noise distributed intentionally as a hair

We can observe in Table 1 that the discrete compactness C_D of the sphere with noise is distant from that of the *Sphere* (0.9410 and 0.9834 respectively). This fact is not true for the common compactness definition

$$C = \frac{{A_s}^3}{36\pi V^2}, \tag{8}$$

where A_s is the surface area, and V the volume of the object (see [1] and [9]). Eq.(8) is equal to 1 for a *Sphere*. The compactness of the voxelized *Sphere* we are presenting, computed with Eq.(8) is 1.8642, whereas it is 3.2911 for the sphere with noise. Measure 1/C gives 1.5364 and 0.3038 for the two previous objects, which can be compared with the normalized measure C_D.

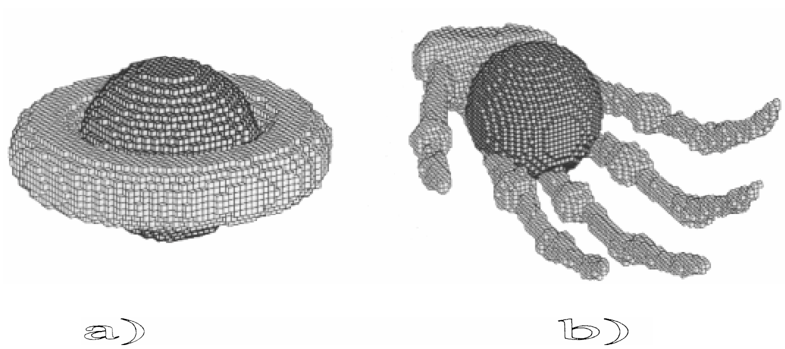

Fig. 5. a) The *Torus* is out of the *Sphere*, there are only four voxels superimposed, b) there are 896 voxels superimposed between *Hand* and *Sphere*

Then, the common compactness measure shows that compactness of a sphere with noise departs significantly from that of a normal sphere. Therefore, the discrete compactness measure, given by Eq.(1), results more robust with regard to the noise.

We should observe in Figure 5 that the voxel distribution of the *Hand* is very different in comparison with the distribution of the Torus3. Even though *Torus3* is out of the *Sphere*, and as a consequence needs to move all its voxels, makes it more similar to the *Sphere* than the *Hand*. As it can be noted, not only the number of positive voxels determine the shape of objects but also the way they are distributed, what is shown in the work done in transformations, and in the similarity between shapes.

Also, it can be noted that compactness is not determinant to establish similarity measure between objects. In fact there is not a linear relationship between work done and compactness of objects. Observe that we have calculated compactness from a number of objects distributed in a rank: from 0.7781 to 0.9834. From these discussions we suggest normalize similarity measure with regard to an object to which compactness be as far as possible from the most compact, on the contrary possible false similarities in shapes can appear. For example if we consider a small rank of compactness given by *Sphere*, *Brain* and *Kidney*, compactness difference is larger between *Sphere* and *Brain*, but work done to transform *Kidney* into *Sphere* is higher.

In summary, we should observe that, from the set of objects **H**, the least compact ones are less similar to the *Sphere* than those more compact, even though the latter sometimes have less voxels to move. It is also shown that when some voxels are moved, e.g. (*Sph-noise*), its similitude with respect to the *Sphere* is the largest of all elements in set **H**, showing that the measure is accurate enough with respect to the noise.

The *Hand* needs the largest work to transform it into the *Sphere*, and it corresponds to the least compact object. Since the *Sphere* and the *Hand*, both belonging to **H**, are the two objects having the highest and smallest compactness, respectively, and the *Hand* takes the largest work to transform it into the *Sphere* we can infer that they are the two most dissimilar among all objects of **H**.

4 Normalization of the Similarity Measure

In this section we discuss two methods to normalize the similarity measure and in both of them we use the compactness information to find the two most dissimilar objects from the whole set. The first method considers a complete set of objects which are obtained by all the combinations of *n* voxels. Then the work done between all the pairs can be found, hence the two most dissimilar objects could be selected to normalize the measure. However, it is not computationally feasible to find all the objects to obtain their transforming work. An alternative to this is computing the work between the objects of highest and lowest compactness. As established in [10], for cubic-shaped voxels the most compact object is the cube formed by those voxels, and the least compact are those with the smallest contact area, such as a stick. The second method has some other advantages, since we only need to compare the least and most compact objects of a given set of *N* objects, e.g. the set **H** with $N = 13$ and $n = 9\ 824$, where the number of objects obtained with *n* voxels is much larger than *N*. Then, we need to find the most and least compact object to compute the two most dissimilar objects of the set **H** to normalize the measure.

4.1 Complete set of objects

The objects that have small work done to transform into the *Sphere* tend to be more similar to the *Sphere* than those with higher work done. This observation evokes the hypothesis of Haralick and Shapiro: “pattern measurements of a given class are nearer to other pattern measurements in the class than to the pattern measurements of other classes” [12].

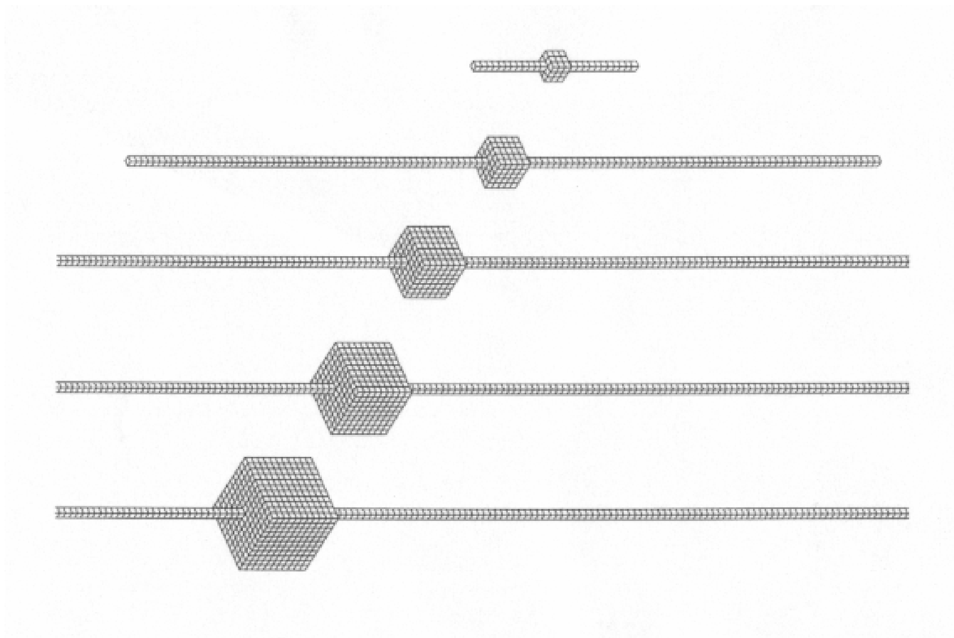

Fig. 6. A cube transformed into a stick at different resolutions

Of course, compactness is not a determinant feature of objects related to similarity. Remember that the number of positive voxels plays an important role as well. By considering a set of *N* objects composed of all possible combinations of *n* voxels, it is not necessary to find all the shapes in order to find those most dissimilar (by the work done in transforming every pair of objects). Clearly, the complete set of objects results huge, in comparison with the 13 objects of the set **H**.

Table 2. Number of positive voxels and work done to transform a stick into a cube at different resolutions

n	$V(+)$	W
27	24	3 360
125	120	4 908
343	336	3 618
729	720	4 566
1 331	1 320	4 467
2 197	26 967	5 009
9 824	9 802.5828	16 236 626.09

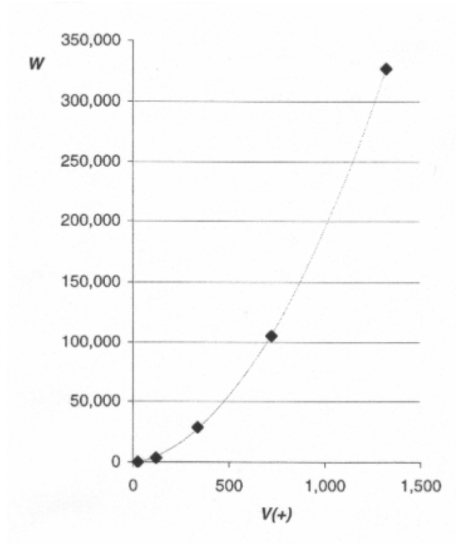

Fig. 7. Work done to transform a box into a stick at different resolutions

From the results of the previous section, we can establish that a procedure to find the two most dissimilar objects from the whole universe of *N* objects composed of *n* voxels, is to transform the most compact object into the least compact object, a cube of $n^{1/3}$ of side, i.e. 1, and a stick of length n, i.e. 0. Figure 6 shows different values of *n* (from up to bottom: 27,125,343,729 and 1331, respectively) to transform a cube into a stick. To know what the work in transforming the cube into the stick composed of *n* voxels is for any n, we could obtain the work for small n's, and then adjust to a polynomial curve. In Table 2 we show the work done between the cube and the stick for five different values of *n*.

Table 3. Number of common voxels and positive voxels to move in each transformation

Object	A_c	A_s	C_D	*Voxels to move*	*Similarity (%)*
Sphere	27 792	3 360	0.9834	0	100
Sph-noise	27 018	4 908	0.9410	419	99.25
Sph-hair	27 663	3 618	0.9763	419	96.66
Brain	27 189	4 566	0.9504	2 385	88.84
Heart	27 238	4 467	0.9531	2 255	87.52
Torus1	27 436	4 701	0.9639	3 706	81.54
Lung	26 967	5 009	0.9382	3 587	81.07
Liver	26 841	5 262	0.9313	3 817	77.74
Kidney	27 480	3 984	0.9663	4 278	72.99
Intest	24 888	9 168	0.8244	6 516	61.58
Torus3	26 773	5 397	0.9276	9 820	46.92
Torus2	26 197	6 549	0.8961	9 824	4.39
Hand	24 041	10 802	0.7781	8 928	0

Table 4. Positive voxels

Object	*Sphere*	*Brain*	*Heart*	*Lung*	*Liver*	*Kidney*	*Intestine*	*Hand*
Sphere	0	2 385	2 255	3 587	3 817	4 278	6 516	8 928
Brain	2 385	0	2 092	2 806	2 556	3 318	5 898	8 920
Heart	2 255	2 092	0	3 091	3 329	3 056	5 942	8 495
Lung	3 587	2 806	3 091	0	2 815	3 088	5 853	8 530
Liver	3 817	2 556	3 329	2 815	0	3 112	5 899	8 894
Kidney	4 278	3 318	3 056	3 088	3 112	0	5 466	8 809
Intestine	6 516	5 898	5 942	5 853	5 899	5 466	0	8 103
Hand	8 928	8 920	8 495	8 530	8 894	8 809	8 103	0

Table 5. Table of work done in transforming between organs (including the *Sphere*) according to *TCV* algorithm

Object	*Sphere*	*Brain*	*Heart*	*Lung*	*Liver*	*Kidney*	*Intestine*	*Hand*
Sphere	0.00	24 131.42	26 989.93	40 923.34	48 138.05	58 395.02	83 083.48	216 229.33
Brain	24 131.42	0.00	19 718.00	26 088.59	30 004.38	41 840.54	66 222.82	199 356.66
Heart	26 989.93	19 718.00	0.00	26 217.89	33 014.05	38 005.61	70 126.63	195 251.14
Lung	40 923.34	26 088.59	26 217.89	0.00	25 519.29	31 017.00	65 766.07	186 308.76
Liver	48 138.05	30 004.38	33 014.05	25 519.29	0.00	29 430.51	66 023.26	183 485.76
Kidney	58 395.02	41 840.54	38 005.61	31 017.00	29 430.51	0.00	68 180.42	174 909.38
Intestine	83 083.48	66 222.82	70 126.63	65 766.07	66 023.26	68 180.42	0.00	171 455.71
Hand	216 229.33	199 356.66	195 251.14	186 308.76	183 485.76	174 909.38	171 455.71	0.00

Table 6. Table of work done in transforming between organs (including the *Sphere*) according to Hungarian algorithm

Object	*Sphere*	*Brain*	*Heart*	*Lung*	*Liver*	*Kidney*	*Intestine*	*Hand*
Sphere	0.00	24 102.13	26 104.68	39 538.67	45 420.77	57 395.35	80 050.85	211 077.30
Brain	24 102.13	0.00	17 947.87	23 760.61	27 395.58	40 317.26	62 038.56	192 849.23
Heart	26 104.68	17 947.87	0.00	24 474.99	31 139.70	36 032.00	66 726.50	189 912.71
Lung	39 538.67	23 760.61	24 474.99	0.00	23 181.34	28 983.46	60 433.50	178 730.21
Liver	45 420.77	27 395.58	31 139.70	23 181.34	0.00	27 983.27	61 068.80	172 813.31
Kidney	57 395.35	40 317.26	36 032.00	28 983.46	27 983.27	0.00	65 035.87	165 670.62
Intestine	80 050.85	62 038.56	66 726.50	60 433.50	61 068.80	65 035.87	0.00	163 495.79
Hand	211 077.30	192 849.23	189 912.71	178 730.21	172 813.31	165 670.62	163 495.79	0.00

We can fit the following equation to this curve: $W = 0.166V^2 + 29.235V$, where $V = V(+)$ is the number of positive voxels. With this second order polynomial we can extrapolate for n = 9 824 voxels, and the work done is approximately 16 237 623, see Figure 7.

Therefore if we normalize with this amount of work (16 237 623) then the *Sphere* and the *Intestine* would be similar in 99.51% and the *Sphere* and the *Hand* in 98.70% .

Of course, these percentages do not correspond to visual perception. Thus, it is more convenient to consider another method that we propose in the next section.

4.2 A noncomplete set of objects

Given a set of *N* objects, suppose each one is composed of *n* voxels. A method to normalize the similarity measure is to look for the most dissimilar pair between the *N* objects. With this amount of work between the most dissimilar objects we have the maximum work done in the transformation of these *N* objects. Considering the objects of the set **H**, it is observed that the two most dissimilar are the *Hand* and the *Sphere*. They also correspond to the objects with the largest and the smallest compactness. Now, by considering the work done in transforming the *Hand* into the *Sphere* as a similarity measure of 0%, and the work done in transforming the *Sphere* into itself as a 100%, we can derive a normalized expression for the similarity degree between two objects,

$$S_{1,2} = \left(1 - \frac{W_{1,2}}{W_{max}}\right) \times 100, \quad (9)$$

where $W_{1,2}$ is the work done in transforming one object into another, and $W_{\max}$ is the maximum work done of the *N* objects transformed. Then, we obtain a table with the work normalized in transforming all the objects into the *Sphere* (Table 3). Of course, this is only a sample of all the existing possibilities from a complete set composed of 9 824 voxels.

Table 7. Normalized similarity measure between objects according to *TCV* algorithm

Object	*Sphere*	*Brain*	*Heart*	*Lung*	*Liver*	*Kidney*	*Intestine*	*Hand*
Sphere	100	88.83	87.52	81.07	77.74	72.99	61.58	0.00
Brain	88.83	100	90.88	87.93	86.12	80.65	69.37	7.80
Heart	87.52	90.88	100	87.87	84.73	82.42	67.57	9.70
Lung	81.07	87.93	87.87	100	88.20	85.66	69.59	13.84
Liver	77.74	86.12	84.73	88.20	100	86.39	69.47	15.14
Kidney	72.99	80.65	82.42	85.66	86.39	100	68.47	19.11
Intestine	61.58	69.37	67.57	69.59	69.47	68.47	100	20.71
Hand	0.00	7.80	9.70	13.84	15.14	19.11	20.71	100

In Table 4 we show the positive voxels when comparing all the pair of organs, including the *Sphere*. These are the voxels that result necessary to move for the transformation of objects.

In Table 5 we show the work done for transforming organs according to *TCV* algorithm, while in Table 6 the work done according to Hungarian algorithm is shown. Observe that the amount of work in Table 6 is always smaller or equal than the work needed in Table 5. Finally, in Tables 7 and 8 we show the normalized similarity measure calculated with Eq.(9), according to *TCV* and Hungarian algorithms, respectively.

Table 8. Normalized similarity measure between objects according to Hungarian algorithm

Object	*Sphere*	*Brain*	*Heart*	*Lung*	*Liver*	*Kidney*	*Intestine*	*Hand*
Sphere	100	88.58	87.63	81.27	78.48	72.81	62.08	0.00
Brain	88.58	100	91.50	88.74	87.02	80.90	70.61	8.64
Heart	87.63	91.50	100	88.40	85.25	82.93	68.39	10.03
Lung	81.27	88.74	88.40	100	89.02	86.27	71.37	15.32
Liver	78.48	87.02	85.25	89.02	100	86.74	71.07	18.13
Kidney	72.81	80.90	82.93	86.27	86.74	100	69.19	21.51
Intestine	62.08	70.61	68.39	71.37	71.07	69.19	100	22.54
Hand	0.00	8.64	10.03	15.32	18.13	21.51	22.54	100

5 Importance of Object Center of Mass and Orientation in Performing the Transformations

Many methods for aligning 3D objects exist in literature. Elsen et al. [13] discuss a classification scheme for image matching methods. Besl and Mckay [14] describe a method based on iterative closest point algorithm to registration of 3-D shapes. Brown discusses different registration methods, such as correlation and sequential methods, Fourier methods and point mapping [15]. Bribiesca [7] for example, aligned 3D objects by maximum correlation, computing the work done in transformation. Lohmann [1] discusses some methods to align volumetric images, namely the iterative closest point, principal axes and correlation algorithms so that they overlap as close as possible. To probe our results, we have used principal axes to orientate the set of objects presented in this work.

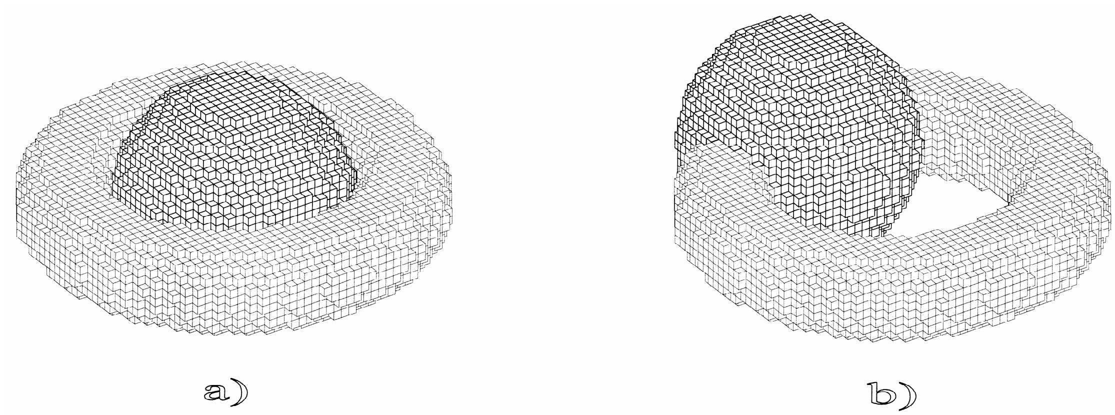

Fig. 8. Transforming *Sphere* into *Torus3*, a) centers of mass of both *Sphere* and *Torus3* coincide and there are only four common voxels; b) their centers of mass are apart by 19 voxel units and they have 2181 common voxels

In this Section we show that, as part of the transformation of objects, superimposing them in such a manner that their centers of mass coincide, and their principal axes are also in the same direction, the work done is minimized. It is also experimentally shown that for some arbitrary orientations, including those where greater number of voxels coincide in superimposition, the work done in transformation increases too. This confirms, in part, some results found in Figure 3, where it can be seen that despite some objects compared with *Sphere* have a larger number of voxels in common, the work for transformation is bigger. In addition, by moving centers of mass and principal axes the work for transformation increases, and in such cases the measure is not optimum.

Experimentally, we have found that transforming *Sphere* into one of the torus, e.g. *Torus3*, requires more work when displacing their centers of mass 19 voxel units (see Figure 8). Hence, the work done to transform objects with different center of mass is larger than that given when centers of mass coincide. When displacing mass center 19 voxel units, then 2 181 voxels match between *Sphere* and *Torus3* giving 44% more work in transformation than when mass centers coincide. On the other hand, displacing centers of mass between *Hand* and *Brain* increases the work, resulting larger when creating a 90^o angle between their largest axes with 2 703 common voxels, with work employed in the transformation of W = 241 711.88. Although, if the objects are aligned as their principal axes and common center of mass, the work done is W = 199 356.66 reduced 20%. See Figure 9 and Table 9 that summarizes these observations.

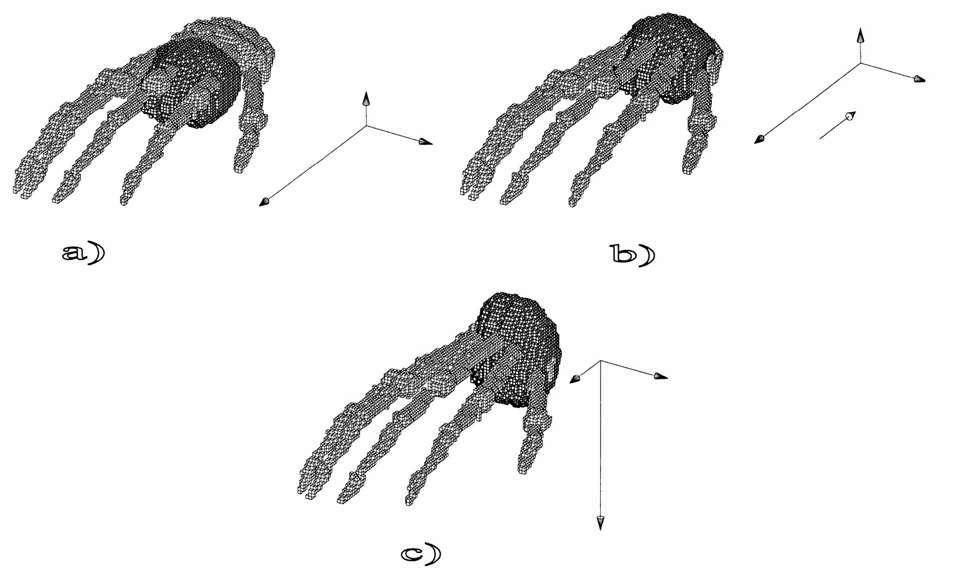

Fig. 9. Transforming *Hand* into *Brain*: a) centers of mass of both objects coincide and there are 904 common voxels; b) their centers of mass are apart by 20 voxel units and they have 3 691 common voxels; and c) their centers of mass are apart of 22 voxel units and they have 2 703 common voxels and major principal axes make 90^o between them

Table 9. Relationship between principal axes and mass centers; common voxels and work done when transforming objects

Object to transform	Transformed object	Angle between greatest axes	Separation between mass centers (voxel units)	$V(+)$	W
Sphere	*Torus3*	$\sim 0^0$	~ 0	9 820	113 735.96
Sphere	*Torus3*	$\sim 0^0$	19	7 643	203 313.93
Hand	*Brain*	$\sim 0^0$	~ 0	8 920	199 356.66
Hand	*Brain*	$\sim 0^0$	20	6 133	209 593.35
Hand	*Brain*	90^0	22	7 121	241 711.88

6 Discussions and Results

The reason being that there are many objects that have capricious shapes not shown here, but that can be formed with the 9 824 voxels as well. Because of this, it results more appropriate to compute the normalization in this manner when we have a larger representative sample of objects with different shapes. Thus, in order to normalize the similarity measure for a sample like the set **H** given in Section 3,

An important hypothesis has arisen from experiments made in this work and may be proved mathematically as a future work.

Hypothesis: Let S_1 and S_2 be two objects composed of n voxels. The minimal work employed to transform the two objects

occurs when their center of mass coincide and are aligned by means of principal axes, if and only if, Hungarian algorithm is applied to match positive voxels to the negative set.

We show in Table 6 the work done to compare all pair of objects, and we have found that the two objects with the largest work for their transformation are the *Sphere* and the *Hand*. This fact is consistent with the result shown in Figure 3, where the objects with the largest and smallest compactness of the set **H** are the same objects that require the largest work in their transformation. Of course, appropriate compactness interval is needed. Wider compactness interval, less false similarities should be obtained.

This means that it is enough to observe the results of Figure 3 to find the objects with the largest work, and from this result then normalize the similarity measure.

We should mention that our normalization of the similarity measure is local. However, it is not convenient to normalize globally under this method of transformation of objects. For instance, if we try to normalize globally we are faced to the problem: "given N objects, each composed of n voxels, find the two most dissimilar between them and normalize the measure", which can be restated as: "given N objects, each composed of n voxels, find the pair that requires the maximum work to transform them and normalize the measure".

In Subsection 4.1 we discuss what happens when we apply this idea to all the objects, each composed of n voxels. However, we conclude in Subsection 4.2 that it is not necessary to include all the possible objects composed of n voxels, but only the set of objects that we are interested on to make the comparison, e.g., the set **H**. This is so, since information about the shape of the rest of objects, which are not in the set **H**, is not known, and it affects the measure. This is the main reason why this normalization results to be local.

If we have N objects, there could be $N(N\text{-}1)/2$ comparisons to normalize the similarity measure. If this is carried out for a set of N objects, it is required to spend a lot of processing time. Instead, to avoid making all these comparisons, we find the discrete compactness of each of N objects, we look for the objects with the smallest and highest compactness, then we compute their transforming work and we normalize the measure.

7 Conclusions

In this work we present two main results: (1) we have proposed a method to normalize a similarity measure given in [7]. We have normalized this measure by using discrete compactness and the amount of work needed for object transformations. This normalization allows us to provide a degree of similarity between 0 and 100%. We have applied such a technique to irregular 3D objects and it gives good results which might be useful in recognizing them. The measure can be defined as a percentage in the similarity degree, where 0% represents the similarity of the two most different among the N objects and 100% of the two most similar. And (2) we also have shown that keeping the same center of mass and orientation of objects by means of principal axes, result crucial for the minimization of the work done in transformations.

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