

Design of Linear Phase IIR Filters with Flat Magnitude Response using Complex Coefficients Allpole Filters

Diseño de Filtros IIR de Fase Lineal con Respuesta en Magnitud Plana usando Filtros Todopolo con Coeficientes Complejos

Alfonso Fernandez Vazquez¹ and Gordana Jovanovic Dolecek²

¹ School of Electronic Engineering; Xidian University
No.2 South TaiBai Road; Xian, Shaanxi
710071; P. R. China

afernan@inaoep.mx

² Department of Electronics; Instituto Nacional de Astrofísica, Óptica, y Electrónica, INAOE
Luis Enrique Erro No. 1; Tonantzintla, Puebla
72840; Puebla, Mexico
gordana@inaoep.mx

Article received on March 15, 2007; accepted on September 03, 2007

Abstract

This paper presents a new method for the design of linear phase IIR filters with flat magnitude response. The method is based on the design of flat digital allpole filters with complex coefficients. Depending on the parity of the allpole filter order the resulting IIR filter have either real or complex coefficients. The parameters of the design are the same as in traditional IIR filter design, i.e., passband and stopband frequencies, ω_p and ω_s , passband droop A_p , and stopband attenuation A_s . Several design examples are provided to illustrate the method. In addition, a design of linear phase modified two-band IIR filter banks and a design of stable IIR filter with an improved group delay are presented as two applications of the proposed method.

Keywords: IIR filters, linear phase, allpole filters, allpass filters, filter banks, improved group delay.

Resumen

Este artículo presenta un nuevo método para el diseño de filtros IIR de fase lineal con respuesta en magnitud plana. El método está basado en el diseño de filtros todopolo con respuesta plana y coeficientes complejos. Dependiendo de la paridad del orden del filtro todopolo, los filtros resultantes IIR tienen coeficientes reales o complejos. Los parámetros de diseño son los mismos que en el diseño tradicional de filtros IIR, esto es, frecuencias de paso y rechazo, ω_p y ω_s , y atenuaciones en la banda de paso y rechazo, A_p y A_s . Varios ejemplos de diseño son dados para ilustrar el método. Finalmente, el diseño de bancos de filtros modificado de dos bandas de fase lineal y el diseño de filtros IIR con retardo de grupo mejorado se presentan como dos aplicaciones del método propuesto.

Palabras clave: Filtros IIR, fase lineal, filtros todopolo, filtros pasatodo, banco de filtros, retardo de grupo mejorado.

1 Introduction

A filter $H_0(z)$ has linear phase if (Vaidyanathan, 1993),

$$H_0(z) = cz^{-k}\tilde{H}_0(z), \quad (1)$$

where z^{-k} is the delay, c is the complex constant with unit magnitude and $\tilde{H}_0(z)$ is the paraconjugate of $H_0(z)$, that is, it is obtained by conjugating the filter coefficients and by replacing z with z^{-1} .

It is well known that causal Finite Impulse Response (FIR) filters can be designed to have linear phase. However, Infinite Impulse Response (IIR) filters can have linear phase only in the noncausal case (Mitra, 2006; Vaidyanathan, 1993; Vaidyanathan and Chen, 1998), (the phase response can be 0 or π). It has been recently shown that filters with a linear phase property are useful in the filter bank design and Nyquist filter design (Argenti et al.,

1996; Djokic et al.,1998; Powell and Chau, 1991; Selesnick, 1998; Willson and Orchard, 1994; Zhang et al., 2001, 2000).

As a difference to methods presented in (Djokic et al., 1998;Powell and Chau, 1991;Willson and Orchard, 1994), our goal is to propose here a new technique to design linear phase IIR filter with flat magnitude response based on complex allpole filters. Additionally, the design specification must be the same as in traditional IIR filter design based on analog filters, i.e., the passband and stopband frequencies, ω_p and ω_s , the passband droop A_p , and the stopband attenuation A_s , as shown in Fig. 1.

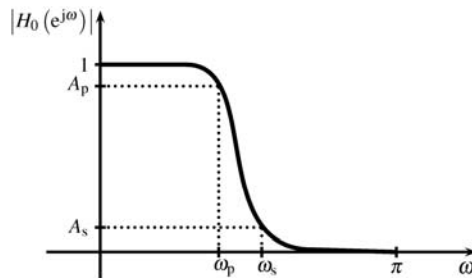


Fig. 1. Design parameters

The challenge with the proposed approach is in how to include the parameters of the design into the desired linear phase IIR filter design. Using results from (Fernandez-Vazquez and Jovanovic-Dolecek, 2004), it follows that an auxiliary complex allpole filter with certain characteristics can be used to solve this problem.

The rest of the paper is organized as follows. In Section 2 we therefore use the known relation between linear phase IIR filter and the corresponding allpass filter. First, we establish design conditions which the allpass filter needs to satisfy, and later we determine design conditions for the auxiliary complex allpole filter. In Section 3, we present closed form equations of the singularities of the corresponding linear phase filter (1) which are used in the design procedures described in Sections 4–6. Detailed description of the algorithms for the lowpass and highpass filters are given in Sections 4 and 5, and are illustrated with different examples. Section 6 presents the applications of the proposed algorithm, including a design of linear phase modified two-band IIR filter banks and a design of a flat IIR filter with an improved group delay.

2 Design of allpole filter with complex coefficients

In this section we design allpole filters with complex coefficients. First, we derive the design conditions that an allpass filter needs to satisfy such that the design of lowpass filter is obtained. Second, to design the corresponding allpole filters, we relate the allpole filter and the allpass filter.

It is well known that a linear phase lowpass IIR filter $H_0(z)$ can be expressed in terms of complex allpass filters as (Zhang et al., 2001),

$$H_0(z) = \frac{1}{2} [A(z) + \tilde{A}(z)], \tag{2}$$

where $A(z)$ is an allpass filter with complex-coefficients. Note that the filter $H_0(z)$ defined in (2) satisfies the relation (1) if $k = 0$ and $c = 1$.

From (2), the magnitude response of $H_0(z)$ can be expressed as,

$$|H_0(e^{j\omega})| = \cos(\phi_A(\omega)), \quad 0 \leq \omega \leq \pi, \tag{3}$$

where $\phi_A(\omega)$ is the phase response of $A(z)$.

Furthermore, we can observe that $|H_0(e^{j\omega})|$ has flat magnitude response at $\omega = 0$, and $\omega = \pi$, and the resulting values $|H_0(e^{j0})|$ and $|H_0(e^{j\pi})|$ equal 1 and 0, respectively.

Since $|H_0(e^{j\omega})|$ has a flat magnitude response at $\omega = 0$, and $\omega = \pi$, the corresponding allpass filter $A(z)$ has a flat phase response at the same frequency points. As a consequence, the resulting group delays $\tau_A(0)$ and $\tau_A(\pi)$ of the allpass filter are equal to 0. Moreover, the values of $\phi_A(\omega)$ at these two frequency points can be 0 and $\pm\pi/2$, respectively.

Considering the value A_p in dB and using (3), it can be shown that the phase value $\phi_A(\omega)$ evaluated at ω_p is given by

$$\phi_{pA} = \phi_A(\omega_p) = \cos^{-1}\left(10^{-A_p/20}\right), \tag{4}$$

In summary, the conditions that the auxiliary complex allpass filter in (2) needs to satisfy, are the following:

- A.1. The phase values of $\phi_A(\omega)$ at $\omega = 0$ and $\omega = \pi$ are 0 and $\pm\pi/2$, respectively (see Fig. 2).
- A.2. The phase response of $A(z)$ is flat at $\omega = 0$ and $\omega = \pi$. Therefore, $\tau_A(0) = \tau_A(\pi) = 0$.
- A.3. The phase value ϕ_{pA} is controlled by A_p (see (4) and Fig. 2).

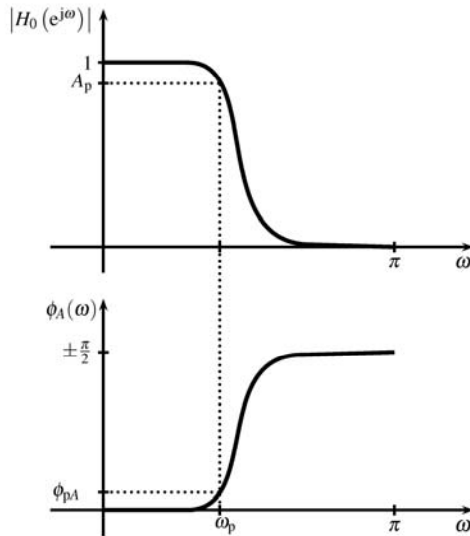


Fig. 2. Relation between the passband droop and the phase value ϕ_{pA}

From the design conditions A.1–A.3, we can note that the complex allpass filter $A(z)$ has causal component $A_c(z)$ and anticausal component $A_a(z)$, such that $A(z) = A_c(z)A_a(z)$. Figure 3 illustrates an implementation of the complex allpass filter. The block TR denotes the time reversal operator, i.e., $r(n) = s(-n)$, and $A_a(z^{-1})$ is now a causal allpass filter (Vaidyanathan and Chen, 1998). The anticausal filter works perfectly in a block by block manner if one correctly chooses initial conditions in the time reverse difference equations (Vaidyanathan and Chen, 1998).

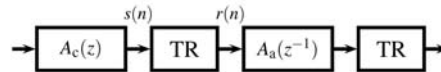


Fig. 3. Structure for the complex allpass filter $A(z)$

In the following we relate a complex allpass filter satisfying conditions A.1–A.3 to the corresponding complex allpole filter.

A complex allpass filter $A(z)$ is related to an allpole filter as follows (Vaidyanathan, 1993),

$$A(z) = z^{-N} \frac{D(z)}{\tilde{D}(z)} = z^{-N} \frac{\alpha \tilde{F}(z)}{\alpha^* F(z)}, \tag{5}$$

where $D(z)$ is an allpole filter of order N given by,

$$D(z) = \frac{\alpha}{F(z)}, \tag{6}$$

the complex constant α has unit magnitude, and $F(z)$ is a polynomial of degree N , that is,

$$F(z) = 1 + \sum_{n=1}^N f_n z^{-n}. \tag{7}$$

The filter coefficients f_n , for $n = 1, \dots, N$, are complex and are expressed as $f_n = f_{Rn} + jf_{In}$, where f_{Rn} and f_{In} are the real and imaginary part of f_n , respectively.

Note that the desired phase of the allpole filter $\phi_D(\omega)$ depends on the phase $\phi_A(\omega)$ of $A(z)$ (see (5)). Therefore, it can be shown that, Selesnick (1999),

$$\phi_D(\omega) = \frac{\omega N + \phi_A(\omega)}{2}. \tag{8}$$

Similarly, the group delay of the allpole filter can be expressed as, Selesnick (1999),

$$\tau(\omega) = \frac{\tau_A(\omega) - N}{2}, \tag{9}$$

where $\tau_A(\omega)$ is the group delay of complex allpass filter.

Using (8) and the phase values $\phi_A(\omega)$ at $\omega = 0$ and $\omega = \pi$ (see Condition A.1), we arrive at $\phi_D(0) = 0$ and $\phi_D(\pi) = (2N \pm 1) \pi/4$.

From the relation (9) and Condition A.2, we further get $\tau(0) = \tau(\pi) = -N/2$.

Finally, the following relation is obtained using Condition A.3 and (8),

$$\phi_D(\omega_p) = \phi_p = \frac{\cos^{-1}(10^{-Ap/20}) + \omega_p N}{2}. \tag{10}$$

As a consequence, the corresponding conditions that the allpole filter $D(z)$ has to satisfy are:

- B.1. The phase values of $D(z)$ at $\omega = 0$ and $\omega = \pi$ are 0, and $\pi(2N \pm 1)/4$, respectively.
- B.2. The group delays $\tau(\omega)$ of $D(z)$ at $\omega = 0$ and $\omega = \pi$ are $-N/2$.
- B.3. The phase value of $D(z)$ at ω_p , $\phi_p = \phi_D(\omega_p)$, is given by (10).

Substituting conditions B.1 and B.2 into the set of linear equations proposed in (Fernandez-Vazquez and Jovanovic-Dolecek, 2004), we arrive at

$$\sum_{n=1}^N \left(n - \frac{N}{2}\right)^k f_{Rn} + \tan(\phi_\alpha) \sum_{n=1}^N \left(n - \frac{N}{2}\right)^k f_{In} = -\left(-\frac{N}{2}\right)^k, \quad k \text{ odd}, \tag{11a}$$

$$\sum_{n=1}^N \left(n - \frac{N}{2}\right)^k f_{Rn} - \cot(\phi_\alpha) \sum_{n=1}^N \left(n - \frac{N}{2}\right)^k f_{In} = -\left(-\frac{N}{2}\right)^k, \quad k \text{ even}, \tag{11b}$$

$$\sum_{n=1}^N (-1)^n \left(n - \frac{N}{2}\right)^k f_{Rn} + \frac{1 + \tan(\phi_\alpha)}{1 - \tan(\phi_\alpha)} \sum_{n=1}^N (-1)^n \left(n - \frac{N}{2}\right)^k f_{In} = -\left(-\frac{N}{2}\right)^k, \quad k \text{ odd}, \tag{11c}$$

$$\sum_{n=1}^N (-1)^n \left(n - \frac{N}{2}\right)^k f_{Rn} + \frac{1 - \tan(\phi_\alpha)}{1 + \tan(\phi_\alpha)} \sum_{n=1}^N (-1)^n \left(n - \frac{N}{2}\right)^k f_{In} = -\left(-\frac{N}{2}\right)^k, \quad k \text{ even}, \tag{11d}$$

where ϕ_α is the phase of α .

Unlike the method of Selesnick (1999), which proposes the design of flat allpole filters with real-value coefficients and different degree of flatness at $\omega = 0$ and $\omega = \pi$, in this method the degree of flatness at $\omega = 0$ and $\omega = \pi$ is $N-2$.¹ It makes that the passband frequency ω_p becomes a continuous variable. Consequently, the values of k are $0, \dots, N-1$.

Solving the set of equations (11) with the symbolic tool MAPLE (Heck, 2003), the filter coefficients are expressed as,

$$f_n = \begin{cases} \binom{N}{n}, & n \text{ even}; \\ \binom{N}{n} (\sqrt{2}e^{j(2\phi_\alpha + \pi/4)} - j), & n \text{ odd}. \end{cases} \tag{12}$$

Finally, from the general phase equation proposed in (Fernandez-Vazquez and Jovanovic-Dolecek, 2004), the following relation gives the desired phase ϕ_p of the allpole filter (Condition B.3),

$$\sum_{n=0}^N \sin(\omega_p n + \phi_\alpha - \phi_p) f_{Rn} - \sum_{n=0}^N \cos(\omega_p n + \phi_\alpha - \phi_p) f_{In} = 0. \tag{13}$$

From (15), the corresponding value of ϕ_α , for N even, is equal to

$$\phi_\alpha = \angle \left\{ -j - 1 - (-1)^{N/2} A_p' \tan^N \left(\frac{\omega_p}{2} \right) \right\}, \tag{14}$$

where $\angle\{\cdot\}$ stands for the angle of $\{\cdot\}$ and

¹ The degree of flatness is defined as the number of null derivatives of the group delay (Fernandez-Vazquez and Jovanovic-Dolecek, 2004).

$$A_p' = \sqrt{\frac{10^{A_p/20} + 1}{10^{A_p/20} - 1}} - 1. \tag{15}$$

Similarly, for odd values of N , we have

$$\phi_\alpha = \angle \left\{ -j - 1 + (-1)^{(N+1)/2} A_p' \tan^N \left(\frac{\omega_p}{2} \right) \right\}. \tag{16}$$

3 Closed form equations for the singularities of $H_0(z)$

In this section, we focus on the computation of the singularities of $H_0(z)$.

3.1 Poles

Substituting (12) into (7) and using the two-polyphase components of $(1 + z^{-1})^N$, we arrive at

$$F(z) = e^{j\phi_\alpha} \left[(\cos \phi_\alpha - \sin \phi_\alpha)(1 + z^{-1})^N - (j - 1) \sin \phi_\alpha (1 - z^{-1})^N \right]. \tag{17}$$

From (17), it can be shown that the corresponding poles of $H_0(z)$ are

$$p_k = \frac{\lambda_k + 1}{\lambda_k - 1}, \tag{18}$$

where $k = 0, \dots, N-1$, and

$$\gamma_k = \left(\frac{\sqrt{2}}{1 - \cot \phi_\alpha} \right)^{1/N} e^{-j \frac{8k+1}{4N} \pi}. \tag{19}$$

3.2 Zeros

It follows from (17) and (2) that the transfer function $H_0(z)$ is,

$$H_0(z) = \frac{(1 + z^{-1})^N E(z)}{z^{-N} F(z) \tilde{F}(z)}, \tag{20}$$

Where

$$E(z) = (1 - \sin(2\phi_\alpha))(1 + z^{-1})^N + c(\cos(2\phi_\alpha) + \sin(2\phi_\alpha) - 1)(1 - z^{-1})^N, \tag{21}$$

and

$$c = \begin{cases} 1, & N \text{ even;} \\ -j, & N \text{ odd.} \end{cases} \tag{22}$$

Observe that the transfer function $H_0(z)$ has N zeros at $z = -1$ and the other zeros at (cf. (21)),

$$z_k = \frac{\beta_k + 1}{\beta_k - 1}, \tag{23}$$

where $k = 0, \dots, N-1$, and the parameter β_k is given by,

$$\beta_k = \begin{cases} \left(\frac{2 \frac{1 - \cos(2\phi_\alpha)}{1 - \sin(2\phi_\alpha)}} \right)^{1/2N} e^{j2\pi k / N} & N \text{ even,} \\ \left(\frac{2 \frac{1 - \cos(2\phi_\alpha)}{1 - \sin(2\phi_\alpha)} \right)^{1/2N} e^{j(4k-1)\pi / 4N} & N \text{ odd.} \end{cases}, \tag{24}$$

Note that there are N zeros away from the origin, which contribute to the passband (see (23) and (24)).

It is easily shown that the absolute values of z_k in (23) for even values of N are always different from 1. However, for N odd, there is one absolute value of z_k equal to 1, i.e., there is a zero on the unit circle. The resulting angle of z_k is denoted by ω_0 . Furthermore, there exists a frequency ω_1 at which $H_0(e^{j\omega})$ equals -1 . Consequently, for odd values of N , the filter $H_0(z)$ has complex-valued coefficients.

Accordingly, the corresponding frequencies ω_0 and ω_1 are expressed as,

$$\omega_0 = \pi + 2 \tan^{-1} \left(2 \frac{1 - \cos(2\phi_\alpha)}{1 - \sin(2\phi_\alpha)} \right)^{1/2N}, \quad \omega_1 = \pi + 2 \tan^{-1} \left(\frac{1}{2} \frac{1 - \cos(2\phi_\alpha)}{1 - \sin(2\phi_\alpha)} \right)^{1/2N}. \tag{25}$$

4 Algorithm for lowpass filter design

In the following we describe the algorithm for the design of linear phase lowpass IIR filters. At first, we compute the allpole filter order N using either (14) or (16).

To satisfy the design condition at ω_s , we replace the passband frequency ω_p with the stopband frequency ω_s and the passband droop A_p with the stopband attenuation A_s in (14). Consequently, the relationship (14) becomes

$$\phi_\alpha = \angle \left\{ -j - 1 - (-1)^{N/2} A_s' \tan^N \left(\frac{\omega_s}{2} \right) \right\}, \tag{26}$$

Where

$$A_s' = \sqrt{\frac{10^{A_s/20} + 1}{10^{A_s/20} - 1}} - 1. \tag{27}$$

Solving (14) and (26) with respect to N , we arrive at

$$N = \frac{\left| \log \left(\frac{A_p'}{A_s'} \right) \right|}{\left| \log \left(\frac{\omega_s'}{\omega_p'} \right) \right|}, \tag{28}$$

where $\lceil \cdot \rceil$ denotes the ceiling function and

$$\omega'_p = \tan\left(\frac{\omega_p}{2}\right), \quad \omega'_s = \tan\left(\frac{\omega_s}{2}\right). \tag{29}$$

As we pointed out before, if the allpole filter order were odd, the resulting filter $H_0(z)$ would have complex-valued coefficients, otherwise it has real-valued coefficients.

The proposed algorithm consists of the following steps.

1. Estimate the order N of the allpole filter using (28).
2. From the values N , ω_p and A_p , compute the phase value of the allpole filter ϕ_α using (14) for N even, and using (16) for N odd.
3. Compute the allpole filter coefficients f_n , using (12).
4. The poles and zeros of the desired filter $H_0(z)$ are computed using (18) and (23), respectively. Additionally, if N is odd, compute the frequencies ω_0 and ω_1 , using (25).
5. Calculate the filter coefficients of $H_0(z)$ using (2).

We illustrate the procedure with the following two examples. The first one uses an even allpole filter order N , whereas the second one use N odd.

Example 1. We design an IIR linear phase lowpass filter with the passband and stopband frequencies $\omega_p = 0.25\pi$ and $\omega_s = 0.45\pi$, respectively. The passband droop is $A_p = 1$ dB, while the stopband attenuation is $A_s = 40$ dB.

1. Using (28), we estimate $N = 8$. Accordingly, the resulting filter $H_0(z)$ has real-valued coefficients.
2. We calculate the phase value $\phi_\alpha = -2.355044$. Therefore, the value of α is $-0.706293 - j0.707920$.
3. The filter coefficients f_n are computed from (12) and are shown in Table 1.

Table 1. Allpole Filter coefficients in Example 1

n	f_n
0	8
1	7
2	6
3	5
4	4

4. Using (23) and (18), we compute the poles and zeros of $H_0(z)$, which are given in Table 2.

Table 2. Poles and zeros of the transfer function $H_0(z)$ in Example 1

Poles and zeros			
$p_1, p_5^*, 1/p_9^*, 1/p_{13}$	$1.966489 - j1.075212$	z_1, \dots, z_8	-1
$p_2, p_6^*, 1/p_{10}^*, 1/p_{14}$	$0.836993 - j0.929015$	$z_9, z_{11}^*, 1/z_{13}^*, 1/z_{15}$	$2.115040 - j0.879601$
$p_3, p_7^*, 1/p_{11}^*, 1/p_{15}$	$0.481181 - j0.492215$	$z_{10}, z_{12}^*, 1/z_{14}^*, 1/z_{16}$	$0.942554 - j0.946345$
$p_4, p_8^*, 1/p_{12}^*, 1/p_{16}$	$0.378588 - j0.127470$		

5. The magnitude response of the designed filter is given in Fig. 4(a) in solid line. For comparison purpose, we design two filters using the methods (Djokic et al., 1998; Powell and Chau, 1991; Willson and Orchard, 1994). These designs are based on traditional Butterworth filter design. Figure 4(a) shows the resulting filters using 8 and 16 zeros at $z = -1$. The passband and stopband details of the proposed filter are shown in Fig. 4(b).

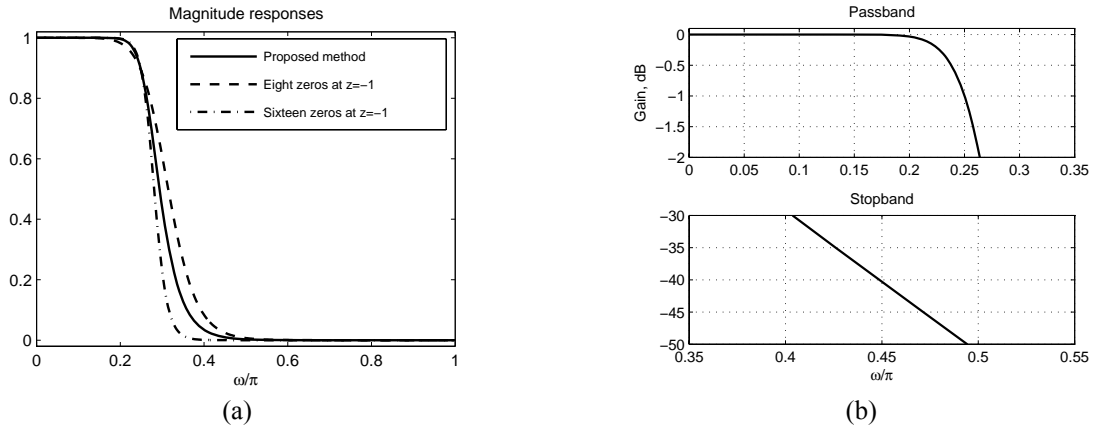


Fig. 4. Magnitude response of $H_0(z)$ in Example 1

Observe that for the same number of zeros at $z = -1$ (8 zeros) the proposed method exhibits better magnitude characteristic.

Example 2. In this example, we illustrate the algorithm for the IIR filter design based on a complex allpole filter with odd order. The IIR filter specification includes the passband and stopband frequencies being 0.5π and 0.7π , and the passband droop and the stopband attenuation being 1 dB and 40 dB, respectively.

1. From (28), it follows that $N = 9$. Therefore, the resulting IIR filter $H_0(z)$ has complex filter coefficients.
2. The phase value ϕ_α is obtained from (18), i.e., $\phi_\alpha = -2.906242$ and $\alpha = -0.972433 - j0.233184$.
3. The filter coefficients of $F(z)$, which are obtained using (12), are shown in Table 3.

Table 3. Filter coefficients of $F(z)$ in Example 2

n	f_n	n	f_n
0	1	5	$55.155296 + j43.439940$
1	$3.939664 + j3.102853$	6	84
2	36	7	$15.758656 + j12.411411$
3	$36.770198 + j28.959960$	8	9
4	126	9	$0.437740 + j0.344761$

4. From (23) and (18), we compute the poles and zeros of $H_0(z)$, which are shown in Table 4. Additionally, the frequencies ω_0 and ω_1 are 1.483715π and 1.459303π , respectively (see (25)), where ω_0 and ω_1 are the frequencies at which $H_0(e^{j\omega})$ is 0 and -1 , respectively.

Table 4. Poles and zeros of $H_0(z)$ in Example 2

Poles and zeros			
$P_1, 1/p_{10}^*$	$-0.045592 - j0.131383$	z_1, \dots, z_9	-1
$P_2, 1/p_{11}^*$	$-0.047014 + j0.221228$	$z_{10}, 1/z_{15}^*$	$-0.025782 - j0.087431$
$P_3, 1/p_{12}^*$	$-0.062954 + j0.635273$	$z_{11}, 1/z_{16}^*$	$-0.031146 - j0.465936$
$P_4, 1/p_{13}^*$	$-0.120521 + j1.296186$	$z_{12}, 1/z_{17}^*$	$-0.378513 + j3.695906$
$P_5, 1/p_{14}^*$	$-0.485810 + j3.102542$	$z_{13}, 1/z_{18}^*$	$-0.077670 + j1.425310$
$P_6, 1/p_{15}^*$	$-11.469800 - j11.130423$	z_{14}	$-0.051140 - j0.998691$
$P_7, 1/p_{16}^*$	$-0.302484 - j2.381486$		
$P_8, 1/p_{17}^*$	$-0.097956 - j1.086518$		
$P_9, 1/p_{18}^*$	$-0.056930 - j0.519239$		

5. Figure 5(a) shows the magnitude response of the designed filter. Figure 5(b) zooms the frequencies ω_0 and ω_1 , while Fig. 5(c) gives the passband and stopband details.

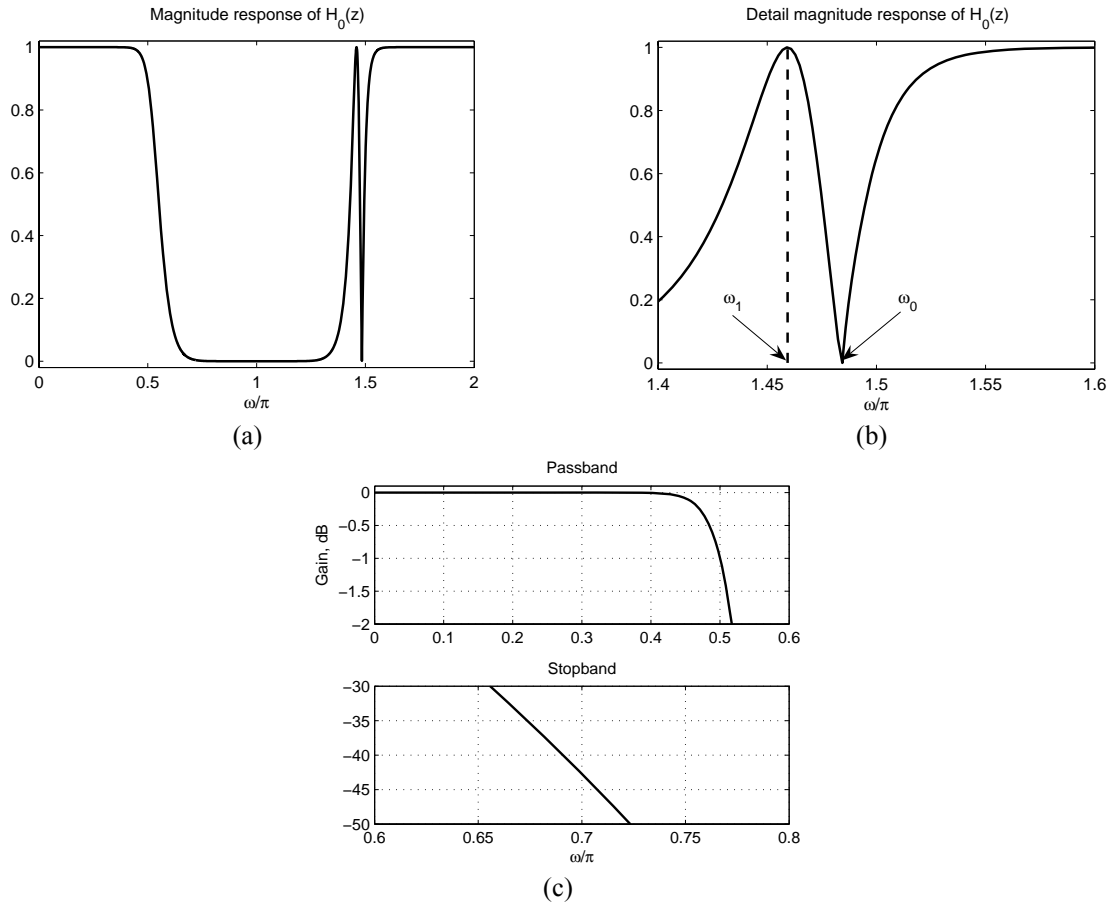


Fig. 5. Magnitude response of the designed lowpass filter in Example 2

5 Algorithm for linear phase IIR highpass filter design

Now, we consider the design of highpass filter using complex-valued allpole filters.

The relationship between a highpass filter and a corresponding allpass filter is expressed by (Vaidyanathan et al., 1987),

$$H_1(z) = \frac{1}{2j} [A(z) - \tilde{A}(z)]. \quad (30)$$

Using (30) and A_p in dB, the phase value ϕ_{pA} is expressed as,

$$\phi_{pA} = \sin^{-1} \left(10^{-A_p/20} \right). \quad (31)$$

The frequencies defined in (29) become

$$\omega'_p = \cot\left(\frac{\omega_p}{2}\right), \quad \omega'_s = \cot\left(\frac{\omega_s}{2}\right). \tag{32}$$

Similarly, the phase value ϕ_α is now given by,

$$\phi_\alpha = \begin{cases} \angle \left\{ -(j+1)\omega'_p - \frac{2(-1)^{N/2}}{A'_p} \right\} & N \text{ even,} \\ \angle \left\{ -(j+1)\omega'_p + \frac{2(-1)^{(N-1)/2}}{A'_p} \right\} & N \text{ odd.} \end{cases} \tag{33}$$

Following example illustrates the design.

Example 3. The parameters of the design of the highpass filter are as follows. The passband frequency $\omega_p = 0.7\pi$ and stopband frequency $\omega_s = 0.4\pi$. The stopband attenuation and passband droop are 45 dB and 2 dB, respectively.

The resulting filter order is equal to 6. The magnitude response, the passband and stopband details of the designed filter are shown in Fig. 6.

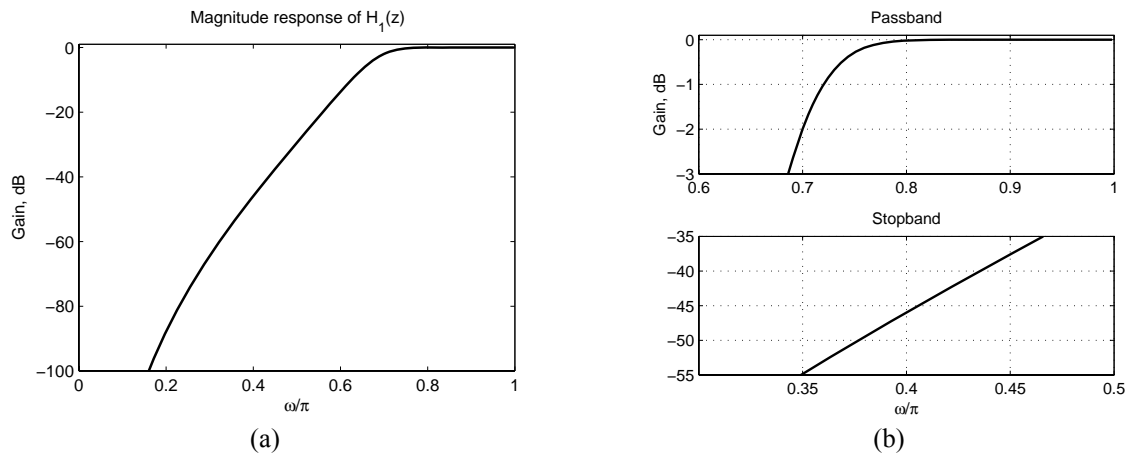


Fig. 6. Magnitude response of $H_1(z)$ in Example 3

6 Applications

This section consider two applications of the proposed design of linear phase IIR filter, i.e., design of modified two-band filter banks and the design of IIR filter with improved group delay.

6.1. Design of Modified Two-Band Filter Banks

The modified two-band filter bank (Galnad and Nussbaumer, 1984), is shown in Fig. 7. The analysis filter $H_0(z)$ and the synthesis filter $G_0(z)$ are lowpass filters, while the analysis filter $H_1(z)$ and the synthesis filter $G_1(z)$ are highpass filters. The difference from the traditional structure is manifested in two extra delays, one before the filter $H_1(z)$ and another after the filter $G_0(z)$ (see Fig. 7).

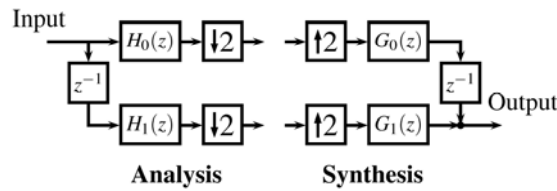


Fig. 7. Modified two-band filter bank

To avoid alias distortion, the synthesis filters are related to the analysis filter $H_0(z)$ in the following form (Vaidyanathan et al., 1987),

$$G_0(z) = 2\tilde{H}_0(z), \quad G_1(z) = 2H_0(-z). \tag{34}$$

where $\tilde{H}_0(z)$ is the paraconjugate of $H_0(z)$, and $H_1(z) = \tilde{H}_0(-z)$.

According to (Vaidyanathan, 1993), the amplitude and phase distortions are eliminated if the analysis filter $H_0(z)$ is chosen to satisfy (Vaidyanathan et al., 1987),

$$H_0(z)\tilde{H}_0(z) + H_0(-z)\tilde{H}_0(-z) = 1. \tag{35}$$

From (35), the relation between the passband droop A_p and the stopband attenuation A_s is given by (Vaidyanathan et al., 1987),

$$10^{-A_p/10} + 10^{-A_s/10} = 1. \tag{36}$$

Additionally, using (34) and (35), we have (Vaidyanathan et al., 1987),

$$\omega_p + \omega_s = \pi. \tag{37}$$

Methods for designing real linear phase modified two-band IIR filter banks based on complex allpass filter are proposed in (Argenti et al., 1996; Zhang et al., 2001; Zhang and Yoshikawa, 1999).

The analysis filters are given by (Argenti et al., 1996),

$$H_0(z) = \frac{1}{2} [A(z) + \tilde{A}(z)], \tag{38}$$

$$H_1(z) = \frac{1}{2j} [A(z) - \tilde{A}(z)], \tag{39}$$

where $A(z)$ is a complex allpass filter.

In (Argenti et al., 1996; Zhang and Yoshikawa, 1999), the corresponding analysis IIR filters are real and have a flat magnitude response in both passband and stopband. More general design of (Zhang and Yoshikawa, 1999) is given in (Zhang et al., 2001) and includes flat as well as equiripple design. In both cases the designed filters are real.

The perfect reconstruction condition for the modified two-band IIR filter banks is established in (Argenti et al., 1996; Vaidyanathan et al., 1987; Zhang et al., 2001; Zhang and Yoshikawa, 1999), which implies that the poles of $H_0(z)$ and $H_1(z)$ must appear on the imaginary axis and in pairs jp and $1/jp$, where p is a pole. From this condition, it follows that the filter coefficients given in (12) must be imaginary for even values of n .

Consequently, the values of ϕ_α in (14) for an even N , must be

$$\phi_\alpha = \begin{cases} -\frac{7}{8}\pi & \text{for } N/2 \text{ even,} \\ -\frac{3}{8}\pi & \text{for } N/2 \text{ odd.} \end{cases} \quad (40)$$

The corresponding zeros of $H_0(z)$ are computed from (23) and (40)

$$z_k = \begin{cases} \frac{1}{2^{2N}} e^{j\pi \frac{2k+1}{N}} + 1 & N/2 \text{ even,} \\ \frac{1}{2^{2N}} e^{j\pi \frac{2k+1}{N}} - 1 & \\ \frac{1}{2^{2N}} e^{j\pi \frac{2k}{N}} + 1 & N/2 \text{ odd,} \\ \frac{1}{2^{2N}} e^{j\pi \frac{2k}{N}} - 1 & \end{cases} \quad (41)$$

where $k = 0, \dots, N-1$, while the poles are computed using (see (18)),

$$p_k = \begin{cases} \frac{\pm j}{\tan\left(\frac{8k+5}{8N}\pi\right)} & N/2 \text{ even,} \\ \frac{\pm j}{\tan\left(\frac{8k+1}{8N}\pi\right)} & N/2 \text{ odd.} \end{cases} \quad (42)$$

Similarly, the values of ϕ_α when N odd are,

$$\phi_\alpha = \begin{cases} -\frac{3}{8}\pi & \text{for } N/2 \text{ even,} \\ -\frac{5}{8}\pi & \text{for } N/2 \text{ odd.} \end{cases} \quad (43)$$

The zeros of $H_0(z)$ are now given by,

$$z_k = \begin{cases} \frac{1}{2^{2N}} e^{j\pi \frac{4k+3}{2N}} + 1 & (N+1)/2 \text{ even,} \\ \frac{1}{2^{2N}} e^{j\pi \frac{4k+3}{2N}} - 1 & \\ \frac{1}{2^{2N}} e^{j\pi \frac{4k+1}{2N}} + 1 & (N+1)/2 \text{ odd,} \\ \frac{1}{2^{2N}} e^{j\pi \frac{4k+1}{2N}} - 1 & \end{cases} \quad (44)$$

where $k = 0, \dots, N-1$, and the poles are expressed as,

$$p_k = \begin{cases} \frac{j}{\tan\left(\frac{8k+1}{8N}\pi\right)} \\ \frac{j}{\tan\left(\frac{8k+3}{8N}\pi\right)} \\ \frac{j}{\tan\left(\frac{8k+5}{8N}\pi\right)} \\ \frac{j}{\tan\left(\frac{8k+7}{8N}\pi\right)} \end{cases} \begin{matrix} (N+1)/2 \text{ even,} \\ \\ (N+1)/2 \text{ odd,} \end{matrix} \tag{45}$$

where $k = 0, \dots, N-1$.

Using (25), we arrive at

$$\omega_0 = \pi + 2 \tan^{-1}\left(2^{1/2N}\right), \quad \omega_1 = \pi + 2 \tan^{-1}\left(2^{-1/2N}\right). \tag{46}$$

6.1.1 Description of the algorithm

In the following we describe how the proposed algorithm can be used for a design of linear phase modified two-band IIR filter banks. The algorithm has the following steps:

1. Calculate the order N of the allpole filter using (28), (36) and (37).
2. Depending on the parity of N , compute the filter coefficients f_n using (12) and either (40) or (43). The corresponding poles and zeros are obtained using (42) and (41) for N even. For N odd use (45) and (44).
3. Additionally, if N is odd, compute the frequencies ω_0 and ω_1 using (46).

We illustrate the method with the next two examples.

Example 4. Passband frequency ω_s of the analysis filter $H_0(z)$ equals 0.6π , whereas the stopband attenuation A_s is 45 dB

1. From (36) and (37), it follows that $A_p = 1.373381 \times 10^{-4}$ and $\omega_p = 0.4\pi$. Using (28), the order of the complex allpole filter is 18.
2. We compute the filter coefficients using (12) and (40), which are shown in Table 5. Poles and zero of $H_0(z)$ are presented in Table 6.

Table 5. Filter coefficients of $F(z)$ in Example 4

n	f_n	n	f_n
0	18	5	13
	1		$-20684.981802j$
1	17	6	12
	$-43.455844j$		18564
2	16	7	11
	153		$-76829.932409j$
3	15	8	10
	$-1969.998267j$		43758
4	14	9	9
	3060		$-117379.063403j$

Table 6. Poles and zeros in Example 4

Poles and zeros			
$p_1, 1/p_{10}^*, p_{19}^*, 1/p_{28}$	j45.829351	z_1, \dots, z_{18}	-1
$p_2, 1/p_{11}^*, p_{20}^*, 1/p_{29}$	j5.027339	$z_{19}, 1/z_{24}^*, z_{28}, 1/z_{32}$	0.009926 + j0.176310
$p_3, 1/p_{12}^*, p_{21}^*, 1/p_{30}$	j2.571496	$z_{20}, 1/z_{25}^*, z_{29}, 1/z_{33}$	0.010902 + j0.363932
$p_4, 1/p_{13}^*, p_{22}^*, 1/p_{31}$	j1.647949	$z_{21}, 1/z_{26}^*, z_{30}, 1/z_{34}$	0.012835 + j0.577279
$p_5, 1/p_{14}^*, p_{23}^*, 1/p_{32}$	j1.140281	$z_{22}, 1/z_{27}^*, z_{31}, 1/z_{35}$	0.016404 + j0.838967
$p_6, 1/p_{15}^*, p_{24}^*, 1/p_{33}$	j0.802585	$z_{23}, 1/z_{36}$	0.009626
$p_7, 1/p_{16}^*, p_{25}^*, 1/p_{34}$	j0.548619		
$p_8, 1/p_{17}^*, p_{26}^*, 1/p_{35}$	j0.339454		
$p_9, 1/p_{18}^*, p_{27}^*, 1/p_{36}$	j0.153915		

Figure 8 shows the corresponding magnitude responses of $H_0(z)$ and $H_1(z)$.

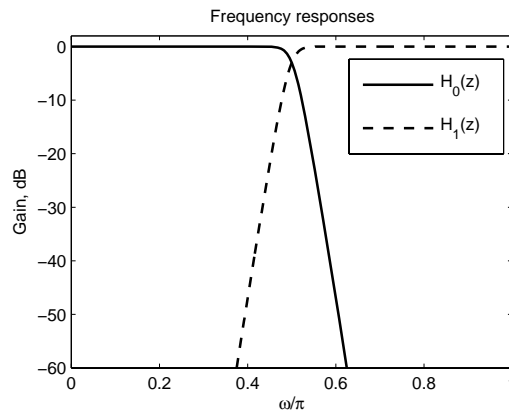


Fig. 8. Magnitude responses of the analysis filters in Example 4

As a difference to the methods (Argenti et al., 1996; Zhang and Yoshikawa, 1999), we give here a closed form equations to compute the IIR filter order and the singularities of $H_0(z)$. Furthermore, the proposed method includes the complex case.

The following example illustrates the complex case (N is odd).

Example 5. We design an IIR filter banks with the following specifications for the analysis filters $H_0(z)$: stopband frequency $\omega_s = 0.8\pi$ and stopband attenuation $A_s = 60$ dB.

1. The estimated value of N is 7.
2. The allpole filter coefficients, and poles and zeros are shown in Tables 7 and 8, respectively. The magnitude response of $H_0(z)$ is shown in Fig. 9.

Table 7. Allpole filter coefficients in Example 5

n	f_n	N	f_n
0	0.38268 + j 0.92388	4	13.39392 + j32.33578
1	15.61300 - j6.46716	5	46.83929 - j19.40147
2	8.03635 + j19.40147	6	2.67878 + j 6.46716
3	8.06549 - j32.33578	7	2.23044 - j 0.92388

Table 8. Poles and zeros of $H_0(z)$

Poles and zeros			
$p_1, 1/p_8^*$	$j17.80665$	z_1, \dots, z_7	-1
$p_2, 1/p_8^*$	$j1.80936$	$z_8, 1/z_{11}^*$	$0.22576 - j2.84187$
$p_3, 1/p_8^*$	$j0.70954$	$z_9, 1/z_{12}^*$	$0.02506 + j0.11260$
$p_4, 1/p_8^*$	$j0.16991$	$z_{10}, 1/z_{13}^*$	$0.03451 + j0.62780$
$p_5, 1/p_8^*$	$-j0.28810$	z_{14}	$0.04947 - j0.99877$
$p_6, 1/p_8^*$	$-j0.89365$		
$p_7, 1/p_8^*$	$-j2.41421$		

3. From (46), we have $\omega_0 = 1.51575\pi$ and $\omega_0 = 1.48425\pi$ (see Fig. 9).

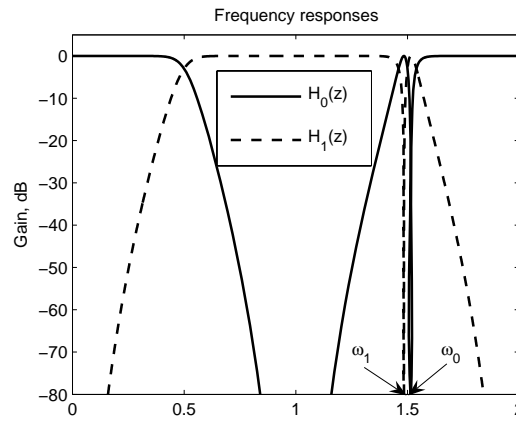


Fig. 9. Magnitude responses in Example 5

6.2. Stable and causal flat IIR filters with an improved group delay

In this section, we relate the linear phase filter with the corresponding causal IIR filters.

Using (1) and (2), we rewrite the relation for a linear phase IIR filter $H_0(z)$ as,

$$H_0(z) = \tilde{H}_0(z) . \tag{47}$$

Considering that the Fourier transform of $H_0(z)$ is real and positive for all ω , we have

$$H_0(z) = H_c(z)\tilde{H}_c(z) , \tag{48}$$

where $H_c(z)$ is a causal and stable IIR filter.

Note that the condition (48) is satisfied using (2) for even values of N . Therefore, we design the filter $H_c(z)$ from the linear phase filter $H_0(z)$.

From (20), it is easily shown that the polynomials $E(z)$ and $F(z)$ are symmetric for even values of N . Consequently, they can be expressed as,

$$E(z) = z^{-N/2}E_0(z)E_0(z^{-1}) , \tag{49}$$

$$F(z) = z^{-N/2}F_0(z)F_0(z^{-1}) , \tag{50}$$

where $E_0(z)$ and $F_0(z)$ are subfilters of $E(z)$ and $F(z)$, respectively.

Substituting (49) and (50) into (20), the transfer function $H_0(z)$ can be rewritten as

$$H_0(z) = \frac{z^{-N/2}(1+z^{-1})^N E_0(z)E_0(-z)}{z^{-N}F_0(z)\tilde{F}_0(z)F_0(-z)\tilde{F}_0(-z)} \tag{51}$$

Using (48) and (51), it follows that

$$H_c(z) = \frac{(1+z^{-1})^{N/2} E_0(z)}{F_0(z)\tilde{F}_0(-z)} \tag{52}$$

Therefore, the transfer function $H_c(z)$ has $N/2$ zeros at $z = -1$.

In order that $H_c(z)$ be stable, all zeros of $F_0(z)$ must be inside the unit circle. There exist different polynomials $E_0(z)$ satisfying (52). The number of polynomials N_{poly} of $E_0(z)$ is expressed as,

$$N_{\text{poly}} = 2^{\lfloor N/4 \rfloor} \tag{53}$$

In the next example, we illustrate how we can get a causal stable IIR filter $H_c(z)$ starting from a linear phase IIR filter $H_0(z)$, using (47) and (52).

Example 6. First, we design an IIR filter $H_0(z)$ with the passband frequency, $\omega_p = 0.25\pi$, and passband droop, $A_p = 1$ dB. The order of the filter is 24.

Consequently, $H_0(z)$ has 12 zeros at $z = -1$, while the remaining poles and zeros are calculated using (23) and (20), respectively. Table 9 shows the poles and zeros.

Table 9. Poles and zeros of $H_0(z)$ when $N = 12$, $\omega_p = 0.25\pi$ and $A_p = 1$

Poles and zeros			
$p_1, 1/p_7$	$0.394884 + j0.139937$	$z_1, 1/z_4$	$0.578620 + j0.590536$
$p_2, 1/p_8$	$0.451398 + j0.374154$	$z_2, 1/z_5$	$0.454151 + j0.339308$
$p_3, 1/p_9$	$0.587022 + j0.634737$	$z_3, 1/z_6$	$0.403978 + j0.110475$
$p_4, 1/p_{10}$	$0.542650 + j0.566504$		
$p_5, 1/p_{11}$	$0.431535 + j0.313685$		
$p_6, 1/p_{12}$	$0.389148 + j0.083698$		

According to (53), there are eight different polynomials for $E_0(z)$, shown in Table 10. The group delays of $H_c(z)$ for all $E_0^l(z)$, $l = 1, \dots, 8$, are shown in Fig. 10.

Table 10. Different polynomials for $E_0(z)$

	z_1, z_1^*	z_2, z_2^*	z_3, z_3^*	z_4, z_4^*	z_5, z_5^*	z_6, z_6^*
E_0^1	×	×	×			
E_0^2	×	×				×
E_0^3	×		×		×	
E_0^4	×				×	×
E_0^5		×	×	×		
E_0^6		×		×		×
E_0^7			×	×	×	
E_0^8				×	×	×

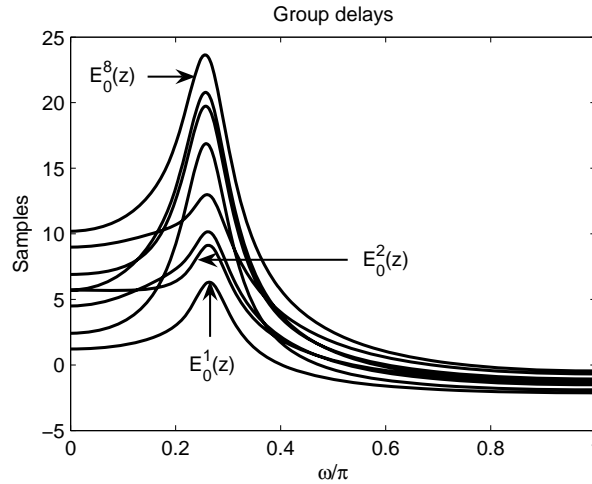


Fig. 10. Different group delays for the IIR filter $H_c(z)$ in Example 6

Figure 10 shows that there is one group delay for each polynomial $E_0^l(z)$, $l = 1, \dots, 8$. We observe that the group delay for $E_0^2(z)$ is less nonlinear in the passband than the others. Therefore, for this example the best polynomial is $E_0^2(z)$.

The next issue we address is how to select the best polynomial for $E_0(z)$ in general.

We have designed many IIR filters and observed that only when the following two conditions are satisfied, the IIR filter with an improved group delay can be obtained.

- The number of zeros of $H_c(z)$ inside and outside the unit circle, N_i and N_o , respectively, are related as

$$N_o \leq N_i, \tag{54}$$

Where

$$N_o = \begin{cases} \lceil N/2 \rceil & \text{if } \lceil N/2 \rceil \text{ has the same parity of } N, \\ \lceil N/2 \rceil - 1 & \text{if } \lceil N/2 \rceil - 1 \text{ has the same parity of } N, \end{cases} \tag{55}$$

and $\lceil \cdot \rceil$ indicates the floor function.

- For each zero z_m inside, and each zero z_l outside of the unit circle, we have,

$$|1/z_l| < |z_m|. \tag{56}$$

6.2.1 Description of the algorithm

The design parameters of the causal stable IIR filters are passband and stopband frequencies, ω_p and ω_s , passband droop A_p , and stopband attenuation A_s . The algorithm has the following steps:

1. We estimate the order of the allpole filter $D(z)$ using results from Section 4,

$$N = \left\lceil \frac{\log\left(\frac{A_p''}{A_s''}\right)}{\log\left(\frac{\omega_s'}{\omega_p'}\right)} \right\rceil, \tag{57}$$

Where

$$A_p'' = \sqrt{\frac{10^{A_p/10} + 1}{10^{A_p/10} - 1}} - 1, \quad A_s'' = \sqrt{\frac{10^{A_s/10} + 1}{10^{A_s/10} - 1}} - 1. \tag{58}$$

If the estimation filter order N is odd, increase it by one.

2. Using the estimated value of N , we calculate the value of ϕ_α as,

$$\phi_\alpha = \angle \left\{ -j - 1 - (-1)^{N/2} A_p'' \tan^N\left(\frac{\omega_s'}{2}\right) \right\}. \tag{59}$$

Compute poles and zeros of $H_c(z)$ in the following form:

- Poles and zeros of $H_0(z)$ are obtained using (23) and (18), respectively.
 - The first N poles of $H_0(z)$ which are inside the unit circle become poles of $H_c(z)$.
 - There are $N/2$ zeros of the filter $H_c(z)$ at $z = -1$ (see (52)). The other $N/2$ zeros are zeros of $H_0(z)$ satisfying conditions (54) and (56).
3. Using the poles and zeros, find the transfer function $H_c(z)$.

Example 7. We design an IIR filter with the following specifications: the passband and stopband frequencies are 0.2π and 0.4π , respectively, the passband droop $A_p = 1$ dB and the stopband attenuation $A_s = 25$ dB.

1. From (59), it follows that $N = 8$.
2. Using the estimated value N and (59), we have $\phi_\alpha = -2.356315$.
3. The resulting poles and zeros of $H_c(z)$ are shown in Table 11, whilst the pole/zero pattern is shown in Fig. 11(a).

Table 11. Poles and zeros of $H_c(z)$

Poles and zeros			
$p_{1,2}$	$0.675540 \pm j0.493940$	$z_{1,\dots,4}$	-1
$p_{3,4}$	$0.617787 \pm j0.416300$	$z_{5,6}$	$1.767678 \pm j0.489703$
$p_{5,6}$	$0.517503 \pm j0.186396$	$z_{7,8}$	$0.661569 \pm j0.442466$
$p_{7,8}$	$0.502595 \pm j0.111476$		

The group delay of the designed filter is shown in Fig. 11(b), while Figs. 11(c) and 11(d) provide the magnitude response.

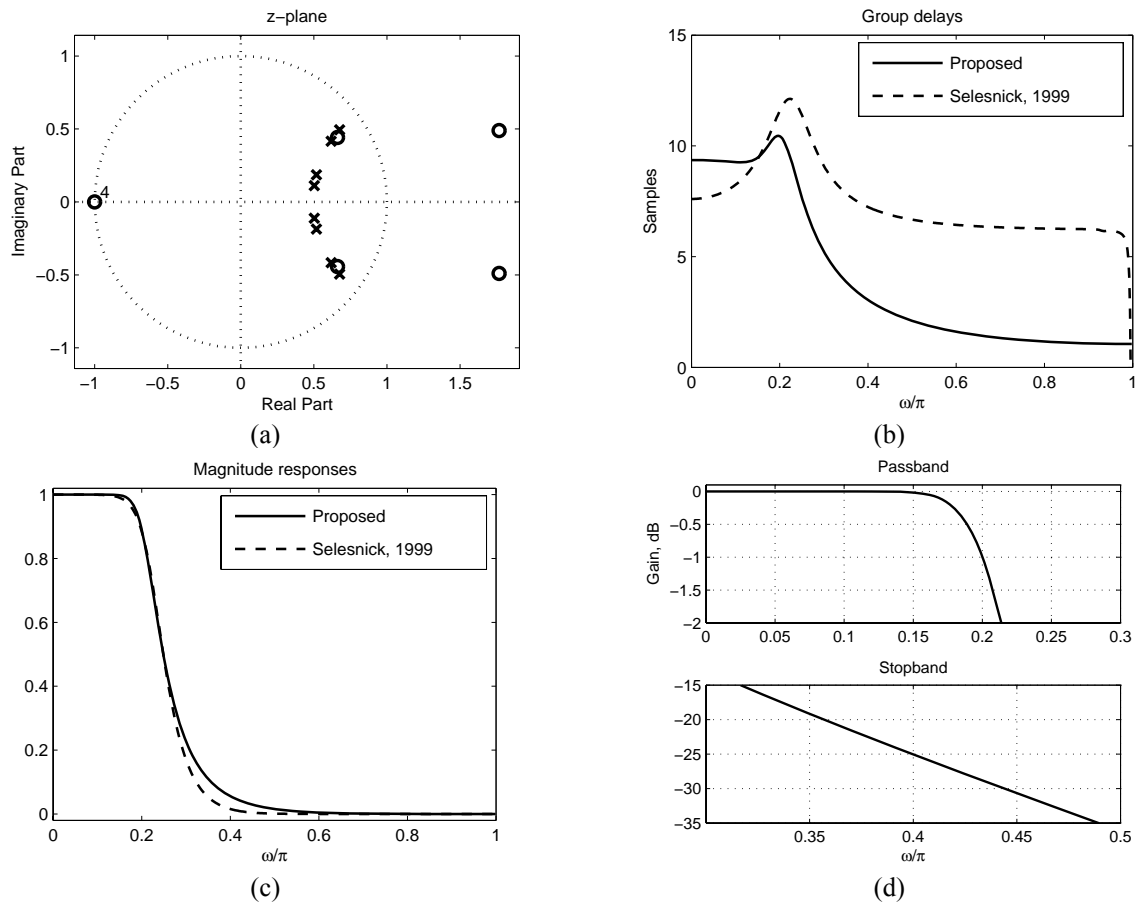


Fig. 11. Example 7

We compare our result with the direct method proposed in (Selesnick, 1999), where the parameters of the design are cutoff frequency ω_c , degree of delay d , and the number of constraints assigned to $\omega = 0$ and $\omega = \pi$, are K and L , respectively. Additionally, the parameter d allows a trade-off between phase-linearity, delay and magnitude response. Using $\omega_c = 0.248925\pi$, $K = 1$, $L = 7$ and $d = 6$, the resulting filter has the degree of the denominator equal to 8. Figures 11(b) and 11(c) show the group delays and magnitude responses of the designed filter using the method (Selesnick, 1999) and the one proposed here.

We observe that the method (Selesnick, 1999) results in an improved overall group delay whereas the proposed filter $H_c(z)$ has a better group delay in the passband.

7 Conclusions

A new method, based on complex-valued allpole filters, for designing real and complex IIR filters with linear phase and flat magnitude response is presented. Since the complex-valued allpole filter can be expressed by the corresponding allpass filter, it follows that, the proposed IIR filter can be efficiently implemented using allpass filters, i.e., the structure based on allpass filters is less sensitivity to filter quantization, Mitra, (2006).

As a difference with others methods, this method is a direct design, i.e., it does not require the design of analog filters and bilinear transformation. Moreover, the parameters of the design are the same as in traditional IIR filter design, that is, passband and stopband frequencies, ω_p and ω_s , passband droop A_p , and stopband attenuation A_s . In contrast to the traditional design of IIR filters, the proposed method can design not only lowpass filters but also highpass filters.

Going to this goal, we establish the conditions that the auxiliary complex allpole filter needs to satisfy in order to design a linear phase IIR filter with the given design parameters. The resulting filter has flat magnitude response in both passband and stopband. Furthermore, closed form equations for the allpole filter coefficients and singularities of the resulting linear phase IIR filter are given. In the proposed filter, N zeros at $z = -1$ control the stopband attenuation while N zeros inside and outside the unit circle contribute to the passband.

Finally, we consider applications of the proposed method. We demonstrate that the new technique can be useful to design linear phase modified two-band filter banks. Other interesting application of the new method, presented in this paper, is the design of stable IIR filters with an improved group delay in the passband.

Acknowledgments

This work was supported by CONACyT Mexico under project number 49640.

References

1. **Argenti, F., Cappellini, V., Sciorpes, A., and Venetsanopoulos, A. N.** "Design of IIR linear-phase QMF banks based on complex allpass sections." *IEEE Trans. Signal Processing*, vol. 44, no. 5, May 1996, pp. 1262–1267.
2. **Djokic, B., Popovic, M., and Lutovac, M.** "A new improvement to the Powell and Chau linear phase IIR filters." *IEEE Trans. Signal Processing*, vol. 46, no. 6, June 1998, pp. 1685–1688.
3. **Fernandez-Vazquez A., and Jovanovic-Dolecek G.** "Design of complex allpass filters," *IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP 2004*, Montreal, Quebec, Canada, May 2004.
4. **Galand, C. R. and Nussbaumer, H. J.** "New quadrature mirror filter structures." *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 32, no. 3, June 1984, pp. 522–531.
5. **Heck, A.** *Introduction to Maple*. Springer-Verlag, New York, third edition, 2003.
6. **Herley, C. and Vetterli, M.** "Wavelets and recursive filter banks." *IEEE Trans. Signal Processing*, vol. 41, no. 8, August 1993, pp. 2536–2556.
7. **Mitra, S. K.** *Digital Signal Processing: A computer based approach*. Mc Graw Hill, third edition, 2006.
8. **Powell, S. R. and Chau, P. M.** "A technique for linear phase IIR filters." *IEEE Trans. Signal Processing*, vol. 39, no. 11, November 1991, pp. 2425–2435.
9. **Selesnick, I. W.** "Formulas for orthogonal IIR wavelet filters." *IEEE Trans. Signal Processing*, vol. 46, no. 4, April 1998, pp. 1138–1141.
10. **Selesnick, I. W.** "Low-pass filter realizable as all-pass sums: Design via a new flat delay filter." *IEEE Trans. Circuits Syst. II*, vol. 46, no. 1, January 1999, pp. 40–50.
11. **Vaidyanathan, P. P.** *Multirate Systems and Filter Banks*. Englewood Cliffs, NJ: Prentice Hall, 1993.
12. **Vaidyanathan, P. P. and Chen, T.** "Structures for anticausal inverses and application in multirate filter banks." *IEEE Trans. Signal Processing*, vol. 46, no. 2, February 1998, pp. 507–514.
13. **Vaidyanathan, P. P., Regalia, P. A., and Mitra, S. K.** "Design of doubly complementary IIR digital filters using a single complex allpass filter, with multirate applications." *IEEE Trans. Circuits Syst.*, vol. 34, no. 4, April 1987, pp. 378–389.
14. **Willson, A. N. and Orchard, H. J.** "An improvement to the Powell and Chau linear phase IIR filters." *IEEE Trans. Signal Processing*, vol. 46, no. 6, June 1994, pp. 2842–2848.

15. **Zhang, X., Kato, A., and Yoshikawa, T.** “A new class of orthonormal symmetric wavelet bases using a complex allpass filter.” *IEEE Trans. Signal Processing*, vol. 49, no. 11, November 2001, pp. 2640–2647.
16. **Zhang, X., Muguruma, T., and Yoshikawa, T.** “Design of orthogonal symmetric wavelet filter using real allpass filters.” *Signal Processing*, vol. 80, no. 8, August 2000, pp. 1551–1559.
17. **Zhang, X. and Yoshikawa, T.** “Design of symmetric orthogonal wavelet filters using a single complex allpass filter.” In *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS'99)*, vol. III. Phoenix, Arizona, March 1999, pp. 367–370.



Alfonso Fernandez Vazquez received the BS degree in Electronics Engineering from Instituto Tecnológico de Puebla in 1998, and MSc and PhD degrees from the Department of Electronics, Instituto Nacional de Astrofísica, Óptica, y Electrónica, INAOE, in 2000 and 2006, respectively. He was a postdoc position at the Universidad de las Américas, Puebla, from July to December 2006. Currently, he is an Assistant Professor at the School of Electronic Engineering; Xidian University; Xian; P. R. China. He is IEEE member and SNI (National Researcher System in Mexico) candidate. His current fields of interest include digital signal processing and digital communications.



Gordana Jovanovic Dolecek received a PhD degree from the Faculty of Electrical Engineering, University of Sarajevo. She was professor at the Faculty of Electrical Engineering, University of Sarajevo until 1993, and 1993-1995 she was with the Institute Mihailo Pupin, Belgrade. In 1995 she joined Institute INAOE, Department for Electronics, Puebla, Mexico, where she works as a professor and researcher. During 2001-2002 and 2006 she was at Department of Electrical & Computer Engineering, University of California, Santa Barbara, as visiting researcher. Her research interests include digital signal processing and digital communications.