Computationally Efficient Multiplier-Free Fir Filter Design

Diseño Eficiente Computacional de Filtros Fir sin Multiplicadores

Gordana Jovanovic Dolecek[1] and Sanjit K. Mitra[2]
[1]Department of Electronics, Institute INAOE, E. Erro 1, Tonantzintla, 72840, Puebla, Mexico
Email: gordana@inaoep.mx
[2]Department of Electriacl & Computer Engineering, University of California Santa Barbara, USA.
Email: mitra@ece.ucsb.edu

Article received on September 01, 2006; accepted on April 12, 2007

Abstract
This paper presents a very simple multiplier-free finite impulse response (FIR) lowpass filter design procedure. It involves approximation of an equiripple FIR by rounding operation and application of the sharpening technique. In that way the overall filter is based on combining one simple filter with integer coefficients. The parameters of the design are the rounding constant and the parameters of the sharpening polynomials such as the order of tangencies *m* and *l*. Our analysis indicates that utilizing this approach the required number of total nonzero bits becomes quite low and less than in the minimum number of signed powers-of-two (MNSPT) design. The cost is the increase of the total numbers of sums and the delays.
Key words: FIR filter, equiripple filter, multiplier-free filter, rounding, sharpening.

Resumen
En este artículo se describe un simple método para diseño de los filtros de pasa baja con la respuesta de impulso finito (FIR) sin multiplicadores. El método consiste de una aproximación del filtro diseñado con el método Remez usando el redondeo y técnica moldeado. De esta manera el filtro deseado se recibe combinando un filtro simple con los enteros coeficientes. Los parámetros de diseño son la constante de redondeo y los parámetros del polinomio moldeado *m* y *l*. Nuestro análisis muestra que necesito numero de bits es bajo y menos que el mínimo numero de bits (MNSPT). El costo es un incremento total de sumas y retrasos.
Palabras clave: Filtro FIR, filtro con iguales rizos, filtro sin multiplicadores, redondeo, moldeado.

1 Introduction

In many applications it is often advantageous to employ finite impulse response (FIR) filters, since they can be designed with exact linear phase and exhibit no stability problems (Mitra, 2006). However FIR filters have a computationally more intensive complexity compared to infinite impulse response (IIR) filters with equivalent magnitude responses. During the past several years, many design methods have been proposed to reduce the complexity of the FIR filters. The main approach is based on optimizing the filter coefficient values such that the resulting filter meets the given specification with its coefficient values represented in minimum number of signed powers-of-two (MNSPT) or canonic signed digits (CSD) representations of binary digits (Bhattacharya and Saramaki, 2003; Lim and Li, 1999; Lim and Liu, 1988; Lim at al., 1991; Kotteri at al., 2003; Wu-Sheng, 2006; Vinod at al., 2003). In general, optimization techniques are complex, can require long run times, and provide no performance guarantees (Kotteri at al., 2003). Some authors have proposed to reduce the number of adders in the multipliers of FIR filters. The common subexpression elimination (CSE) focus on eliminating redundant computations in multiplier blocks using the most commonly occurring subexpressions that exist in the CSD representation (Chang at al., 2006; Winod at al. 2006; Winod at al., 2006; Gentili at al.,1996). Another approach is based on combining simple sub-filters (Vaidanathan and Beitman, 1985; Adams and Williamson, 1983; Jovanovic and Espinosa, 2000; Tai and Lin, 1992; Jovanovic at al., 2005; Bartolo at al. 1998; Jovanovic and Mitra, 2002). Approximation of an equiripple FIR by a rounding operation and implementation of the derived impulse response by a simple recursive equation have also been proposed (Bartolo at al., 1998). In an earlier work (Jovanovic and Mitra,

2002) we used a stepped triangular approximation of the impulse response which can be implemented as a cascade of a recursive running sun (RRS) filter and another RRS filter with a sparse impulse response requiring no multiplications.

The main motivation for this work is to propose simple multiplierless filter design procedure for a desired high performance of the designed filter. We propose a two-step procedure. In the first step, the impulse response coefficients of an equiripple FIR designed to satisfy the given specifications are rounded to the nearest integers. As a difference to (Bartolo at al., 1998) no recursion equation is used. In the next step the sharpening technique (Kaiser and Hamming, 1997; Harnet and Boudreaux, 1995; Donadio, 2003) is applied to the filter with the rounded impulse response such that the given specification is met. As a difference to the method (Gentili at al., 1996), which uses the optimization technique not only for the subfilters but also for sharpening polynomials, we use the fixed simple sharpening polynomials. Methods (Tai and Lin, 1992; Jovanovic at al., 2005) also use the fixed sharpening polynomials but can be applied only for the narrowband filter design. The paper is organized in the following way. In Section 2 and 3 we describe briefly the rounding and the sharpening techniques. The proposed method is described and illustrated with one example in Section 4. Section 5 presents the discussion of the proposed design .

2 Rounding

We use the result proposed in (Bartolo at al., 1998) for the impulse response rounding given as

$$g(n) = \alpha \cdot g_I(n) = \alpha \cdot round(h(n)/\alpha) \quad (1)$$

where $h(n)$ is an equiripple type FIR filter which satisfies given specification, $g_I(n)$ is the new impulse response derived by rounding all coefficients of $h(n)$ to the nearest integer, and *round*(.) means the round operation. The rounded impulse response $g_I(n)$ is scaled by α in order that gain in dB of the rounded filter has the value $(0 \pm R_p)$ dB in the passband, where R_p is the passband ripple. The rounding constant α determines the precision of the approximation of $g(n)$ to $h(n)$. Considering that the integer coefficient multiplications can be accomplished with only shift-and-add operations, the rounded impulse response filter is multiplier-free. Besides the rounding constant is chosen to be in the form $\alpha = 2^{-N}$, where N is an integer.

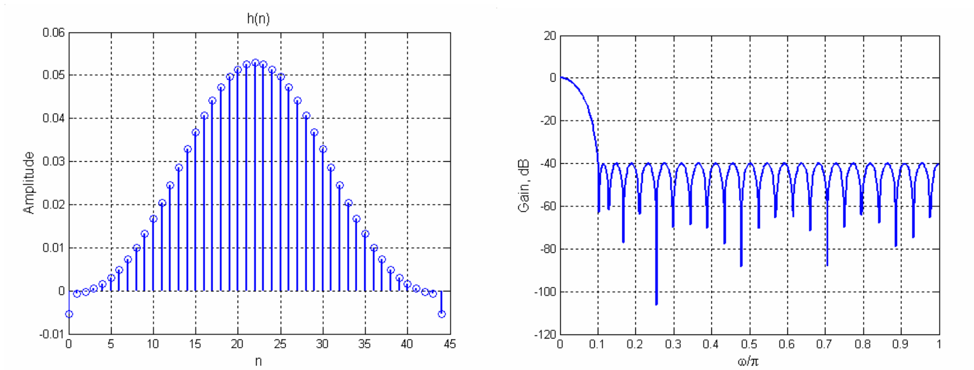
Example 1:

We consider the equiripple filter $h(n)$ satisfying the next specification: the normalized passband and the stopband frequencies ω_p and ω_s are 0.01 and 0.1, respectively, and the passband ripple and the stopband attenuation are R_p=0.2dB and A=40dB, respectively.

Figure 1 shows the impulse response of the equiripple filter and the corresponding gain response in dB. The rounded impulse responses $g_I(n)$, and scaled rounded impulse responses $g(n)$ are shown in Fig. 2. for two values of the rounding constant, α=1/128 (Figures 2.a and b), and α=1/32 (Figures 2.c and d).

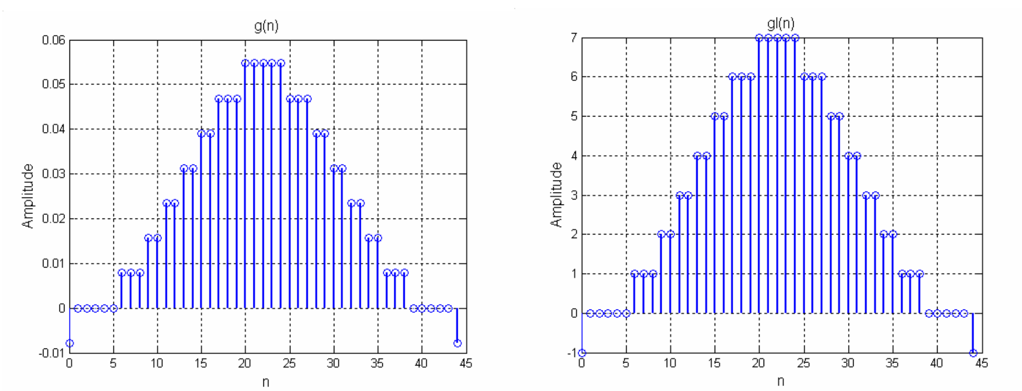
From Figure 2 we can notice:

- The process of rounding introduces some null coefficients in the rounded impulse response (10 in Fig. 2.b, and 20 in Fig. 2.d). The number of nonzero integer coefficients denoted as N_1 corresponds to the number of the sums (35 and 25, for Figures 2.b and 2.d, respectively, and decreases with the increase of the constant α as shown in Figure 3 for the filter in Example 1.

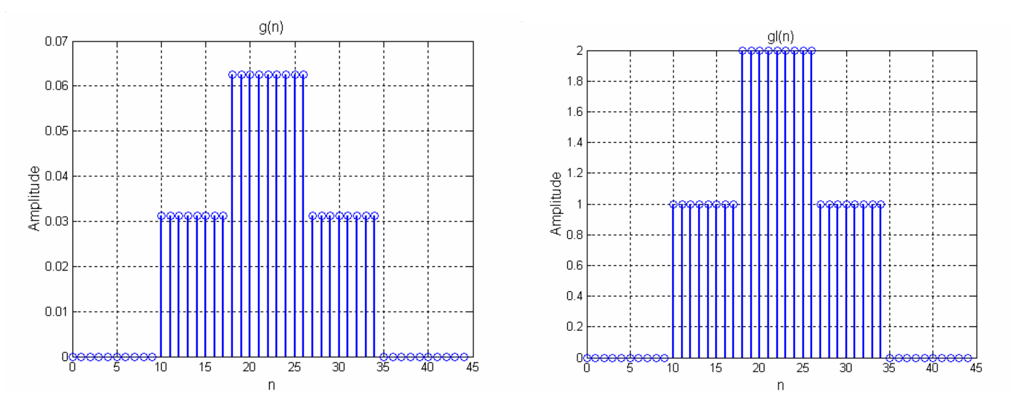


a. Impulse response b. Gain response

Fig. 1. Equiripple filter



a. $g(n)$, α=1/128 b. $g_I(n)$, α=1/128



c. $g(n)$, α =1/32 d. $g_I(n)$, α =1/32

Fig. 2. Scaled rounded and rounded impulse responses for different values of α

- Some of nonzero coefficients of the rounded filter have the same values. Therefore the number of integer multiplications corresponds to the number N_2 of a different positive integer coefficients values. (The values

1 and the corresponding negative values are not counted). The integer multiplications for the rounding constants α=1/128 and α=1/32 are, respectively: 2, 3, 4, 5, 6, and 7, (N_2=6), and 2, (N_2=1). This number also decreases with the increase of the constant α, as shown in Figure 3.

- For the rounded impulse response for α=1/128, the total number of nonzero bits is Nb_1=11, (10 for coefficients and 1 for the rounding constant), while for α=1/32, Nb_1=2, (1 for the coefficient and 1 for the rounded constant). Table 1 shows the values of Nb_1 and the values of Q (maximum number of bits needed to present any coefficient).

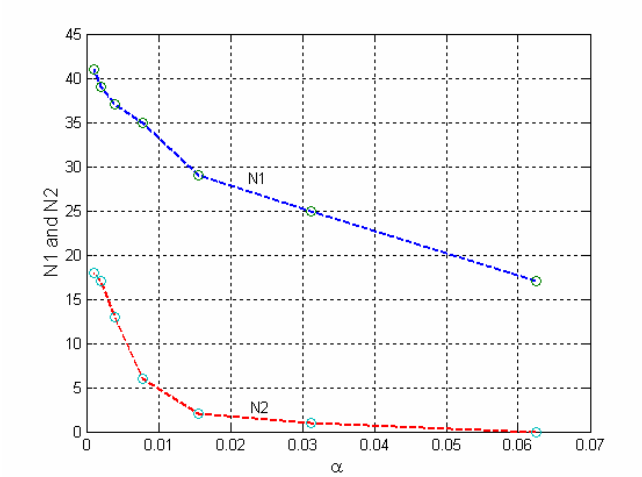
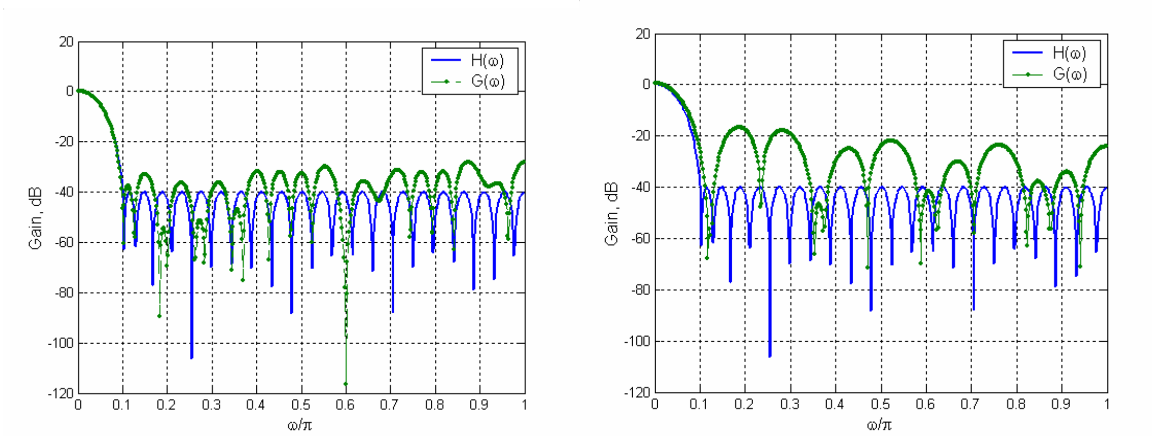


Fig. 3. The numbers of sums (N_1) and integer multiplications (N_2) in Example 1

Table 1. Number of SPT terms for different values of α

Rounded constant *α*	Total number of nonzero bits, Rb_1	Q (maximum number of bits needed to present any coefficient)
2^{-4}	1	1
2^{-5}	2	1
2^{-6}	4	2
2^{-7}	11	2
2^{-8}	25	3
2^{-9}	42	3
2^{-10}	42	4

Figure 4 presents the corresponding gain responses for the filters in Example 1.



a. α=1/128 b. α=1/32

Fig. 4. Gain response of rounded filters

As expected, the rounding alters the original design. The gain response is more altered for a high value of the rounded constant.

Therefore the choice of the rounded constant α must be the compromise between the complexity (less sums and integer multiplications) and less distortion in the desired gain response of the rounded filter. To improve gain response of the rounded filter we propose to use the sharpening technique briefly described in the next Section.

3 Sharpening Technique

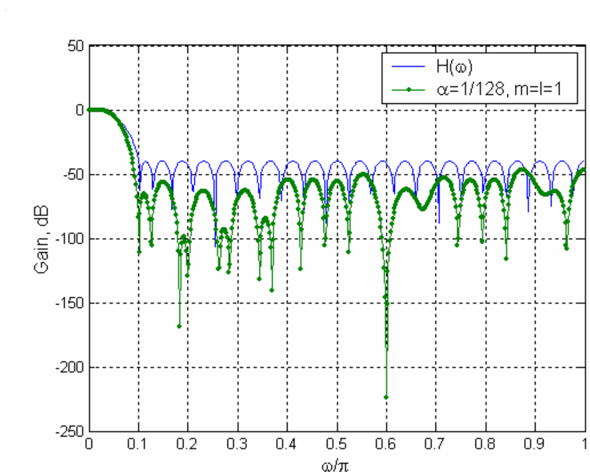
To improve the gain response characteristics, we propose to use the sharpening technique which can be used for simultaneous improvements of both the passband and stopband characteristics of a linear-phase FIR digital filter (Kaiser and Hamming, 1997; Harnet and Boudreaux, 1995; Donadio, 2003). The technique uses the amplitude change function (ACF) which is a polynomial relationship of the form $H_0 = f(H)$ between the amplitudes of the overall and the prototype filters, H_0 and H, respectively. To improve the prototype filter in both passband and stopband the amplitude function has to be horizontal at both $H(z)$=1 (near passband), and $H(z)$=0, (near stopband) i.e. have a derivatives of zero at these points, denoted as m and l, respectively. Kaiser and Hamming, 1997, proposed the following expression for ACF for the given values of m and l ,

$$H_0 = H^{l+1}\sum_{s=0}^{m}\frac{(l+s)!}{l!s!}(1-H)^s \quad = H^{l+1}\sum_{s=0}^{m}C(l+s,s)(1-H)^s, \tag{2}$$

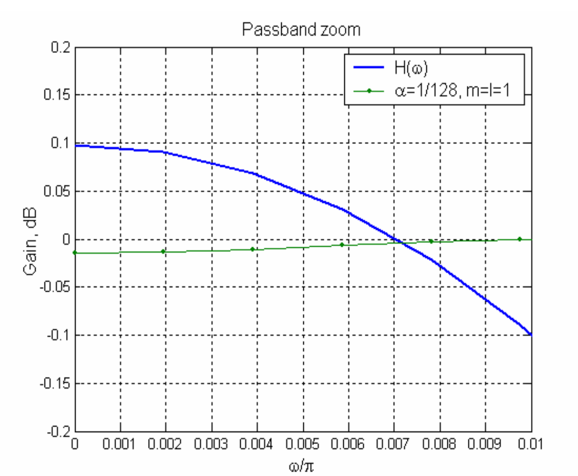
where $C(l+s,s)$ is the binomial coefficient. The values of the ACF for some typical values of m and ℓ , computed from (2) are given in Table II. Figure 5 illustrates the improved magnitude responses of the rounded filters of Figure 4 using different polynomials from Table II.

Table 2. ACF Polynomials for m = 1, 2, 3, and l=1, 2, 3

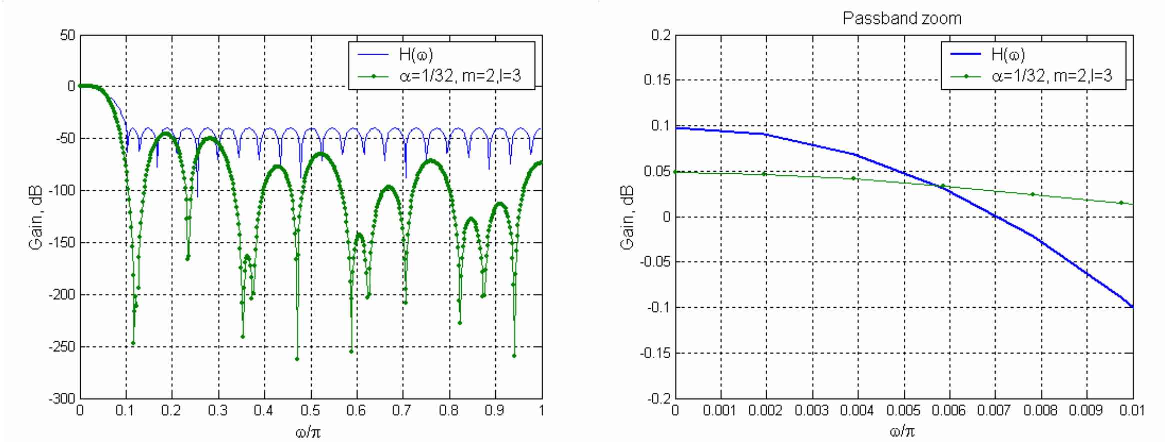
m	l	H_0
0	1	H^2
0	2	H^3
0	3	H^4
0	4	H^5
1	0	$2H-H^2$
1	1	$3H^2-2H^3$
1	2	$4H^3-3H^4$
1	3	$5H^4-4H^5$
1	4	$6H^5-5H^6$
2	0	H^3-3H^2+3H
2	1	$3H^4-8H^3+6H^2$
2	2	$6H^5-15H^4+10H^3$
2	3	$10H^6-24H^5+15H^4$
2	4	$15H^7-35H^6+21H^5$
3	0	$-H^4+4H^3-6H^2+4H$
3	1	$-4H^5+15H^4-20H^3+10H^2$
3	2	$-10H^6+36H^5-45H^4+20H^3$
3	3	$-20H^7+70H^6-84H^5+35H^4$



a. α=1/128, m=l=1



b. α=1/128, m=l=1



a. α=1/32, m=2, l=3 d. α=1/32, m=2, l=3

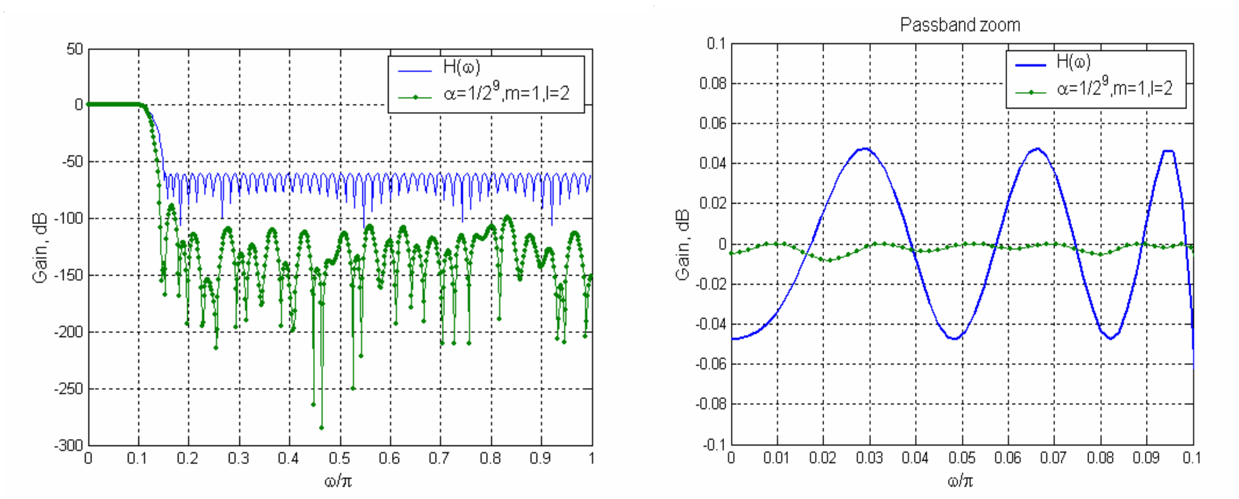
Fig. 5. Sharpened rounded filters in Example 1

4 Filter Design Procedure

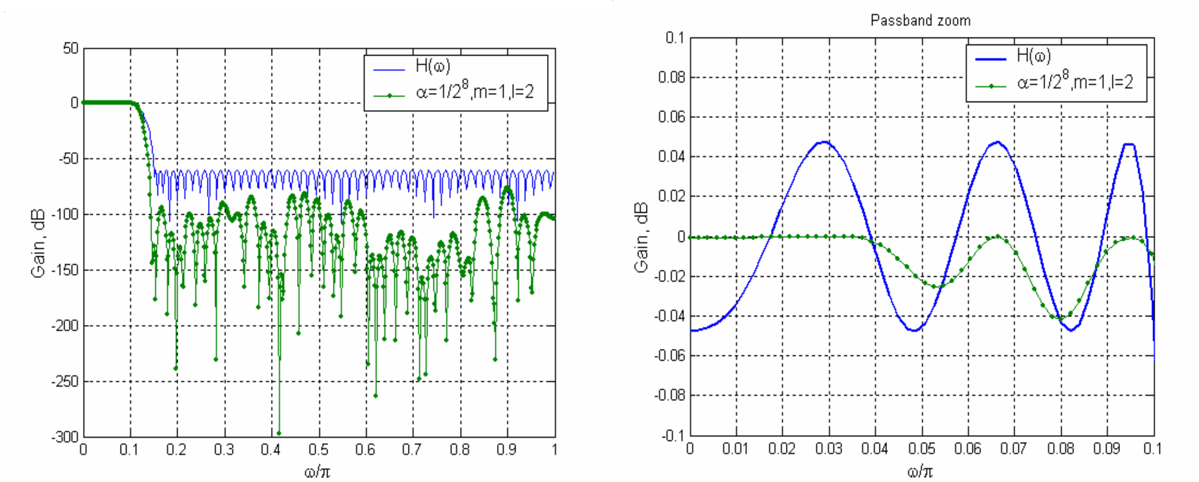
The proposed design procedure is presented in the following steps:
1. In the first step the equiripple filter $h(n)$ satisfying the given specification is designed using Parks-McClelan algorithm.

2. The value of the rounding constant α is chosen in the form $\alpha = 2^{-N}$, where N is an integer. The good starting point for N=6. (See section 2).
3. The coefficients of the filter $h(n)$ are rounded using (1) to obtain filter $g(n)$.
4. Chose the sharpening polynomial and verify if the specification is satisfied. Start with m=1, l=1.
If necessary cascade the sharpened filter or increase the values of m, and l. If the specification is satisfied try to increase the constant α in order to decrease the complexity of the rounded filter. (See Figure 2.)
5. If the specification is not satisfied neither for m=3 and l=3, decrease the rounding constant α and repeat the steps 3 and 4.
The procedure is illustrated with the following example.

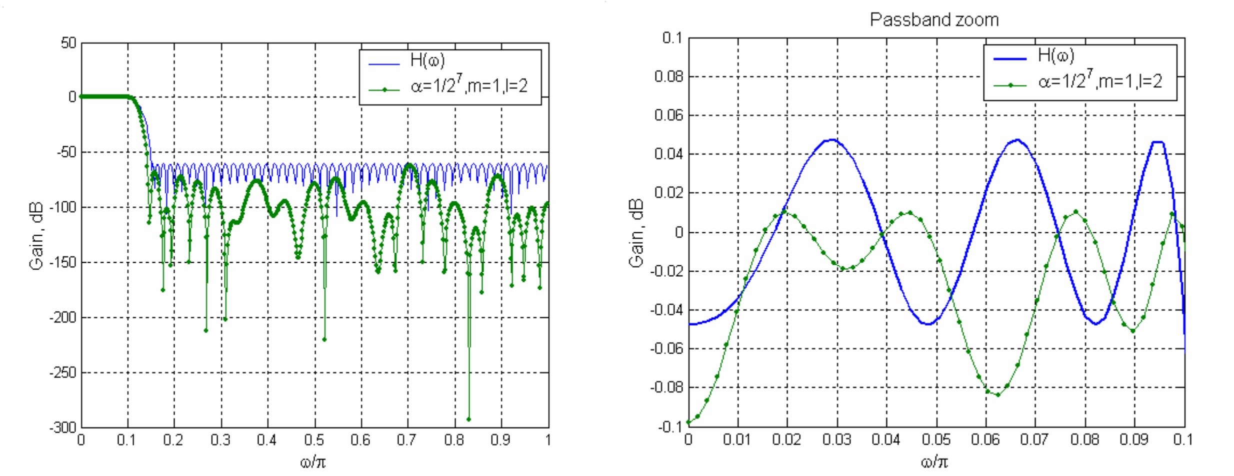
Example 2:
The normalized passband and the stopband frequencies are 0.1 and 0.15, and the passband ripple Rp=0.1dB and the stopband atenuation A=60dB. The length of the equiripple filter N=113. For $\alpha = 2^{-9}$, the number of sums N_1= 89, the number of integer multiplications N_2=15 and m=1, l=2 the specification is satisfied as shown in Fig. 6.a . To decrease the number of integer multiplications we increase the constant α to the value $\alpha = 2^{-8}$ (Fig.6. b), and $\alpha = 2^{-7}$ (Fig.6. c). The resulting values are N_1=75 and N_2=11 for $\alpha = 2^{-8}$ and N_1=53 and N_2=9 for $\alpha = 2^{-7}$.



a. $\alpha = 2^{-9}$



b. $\alpha = 2^{-8}$



c. $\alpha = 2^{-7}$

Fig. 6. Example 2

5 Results And Discussion

We compare the proposed design method with the results of some representative FIR filters (Bhattacharya and Saramaki, 2003), designed using a minimum number s of SPT terms. In (Bhattacharya and Saramaki, 2003) is proposed that one can either accept deviations in the passband and stopband tolerance specifications (passband ripple and the stopband attenuation) compared with the initial infinite-precision design or one can start with a design with stricter specification in order that after quantization the specification is still met.

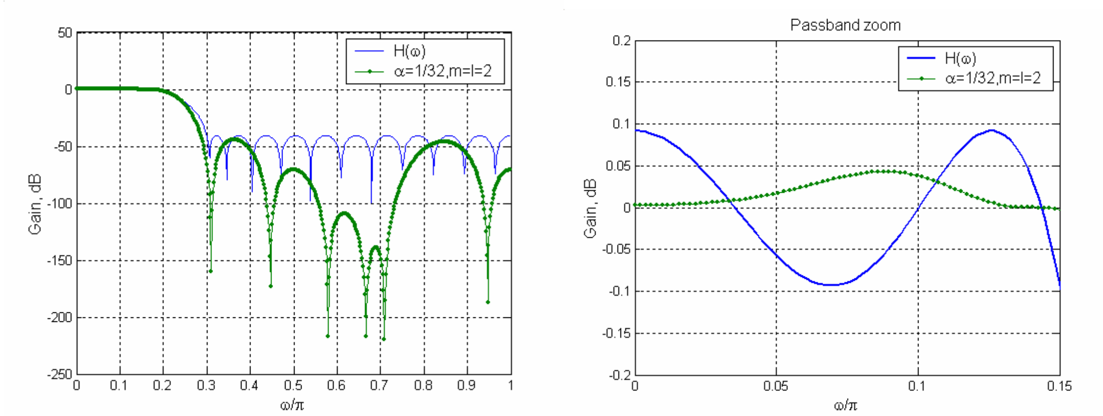
Like in (Bhattacharya and Saramaki, 2003) we consider in more details the case of **Filter 1** with the following specification:

$\omega_p = 0.15\pi\ \omega_s = 0.3\pi, R_p = 0.2dB, As = 40dB.$ The equiripple design results in the filter of the order N=29 and requires 15 multiplications.

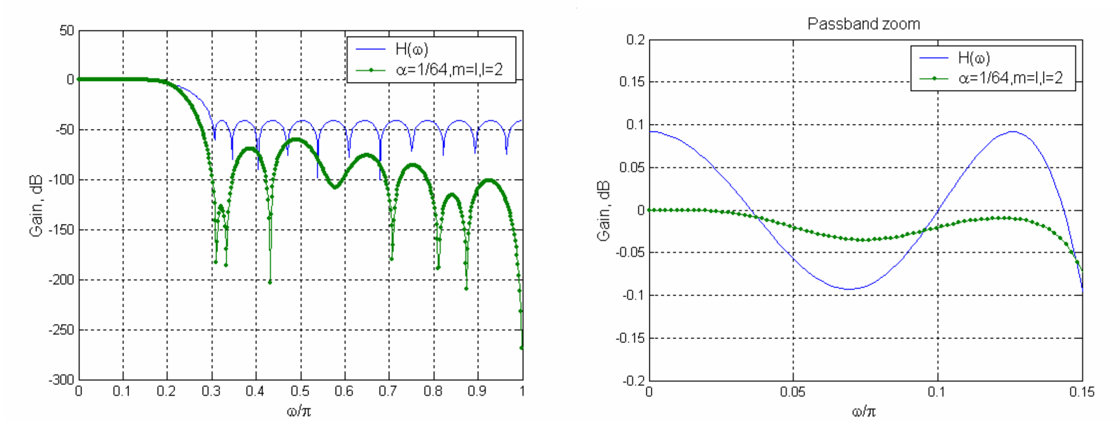
Figure 7 illustrates different designs using different values of α and the parameters m and l.

Table 3 presents the corresponding values of N_1, N_2, m, l, Nb_1, Nb_m and Nb where Nb_1 indicates the total number of nonzero bits for the filter $g(n)$, Nb_m is the average number of nonzero bits per multiplier coefficient, and Nb is the total number of nonzero bits for the multipliers of the designed filter.

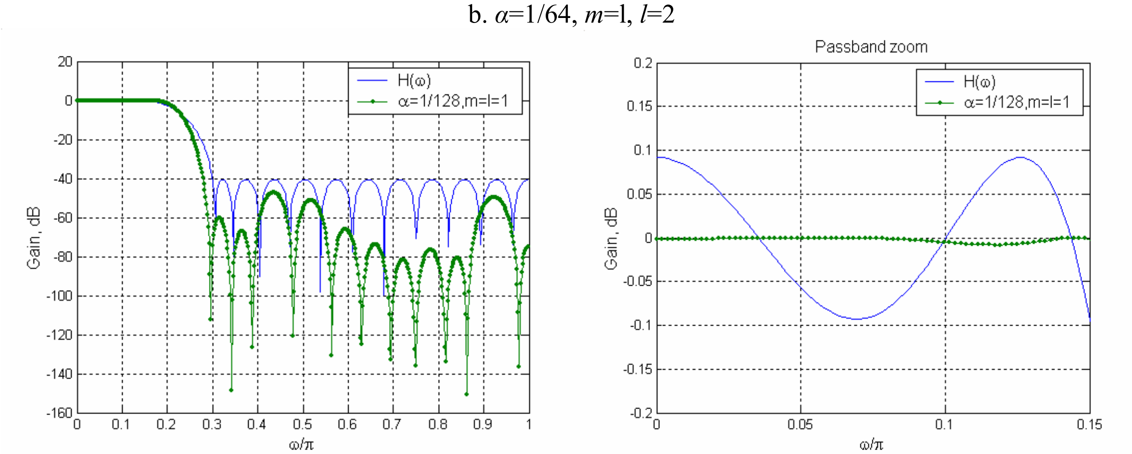
Unlike (Bhattacharya and Saramaki, 2003) we consider here only the cases where the specification is satisfied. The results which are compared with (Bhattacharya and Saramaki, 2003) are presented in bold.



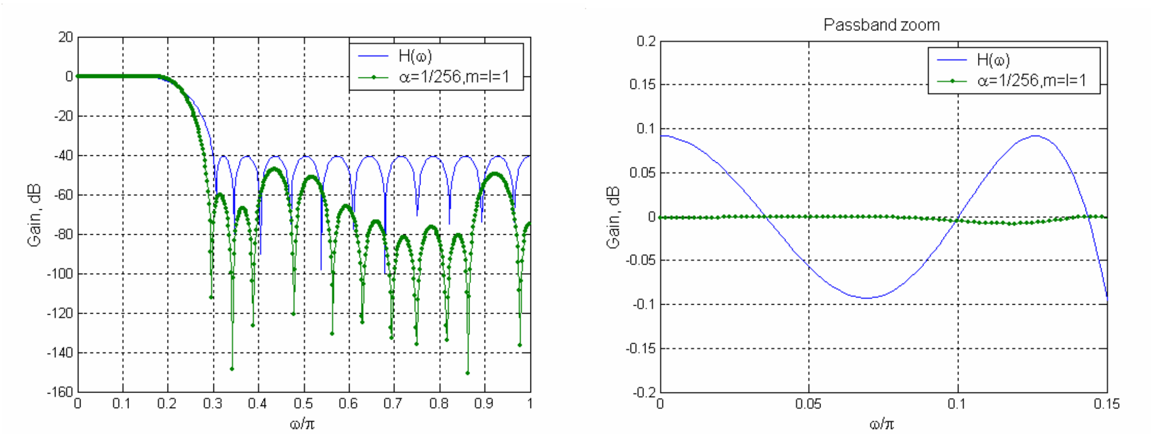
a. α=1/32, m=l=2



b. α=1/64, m=l, l=2



c. α=1/128, m=l=1



d. α=1/256, m=l=1

Fig. 7. Filter 1

Table 3. Results for Filter 1

α	N_1	N_2	m	l	Nb_1	*Nbm*	*Nb*	*R* [dB]	*A*[dB]
1/32	15	3	2	2	6	**2**	**42**	**0.05**	**40.5**
1/64	21	5	1	2	10	**2**	**42**	**0.055**	**60**
1/128	25	8	1	1	17	2.125	52	0.001	42
256	27	11	1	1	24	2.18	73	0.001	42.5
(Bhattacharya and Saramaki, 2003) MNSPT design.						**3.13**	**47**	**0.19**	**40.35**

Note that the average number of nonzero bits per multiplier coefficient *Nbm* , and the total number of nonzero bits for the multipliers *Nb* are less for the proposed design than for MNSPT design, while keeping less passband ripple and higher stopband attenuation. (See first two rows of the Table III).

In the following we consider the revised design (Bhattacharya and Saramaki, 2003) where the filter with the passband ripple *Rp*= 0.15 dB and the stopband specification 45 dB, is designed resulting in the equiripple filter of the order 32. Table IV and Figure 8 illustrate the proposed design for α=1/64 and m=l=1, and the corresponding MNSPT design from (Bhattacharya and Saramaki, 2003), where we can notice that the proposed design is more efficient than the corresponding MNSPT design.

Table 4. Revised design of the Filter 1. (*Rp*=0.15 dB , *A*=45 dB)

α	N_1	N_2	m	1	Nb_1	*Nbm*	*Nb*	*R* [dB]	*A*[dB]
1/64	20	5	1	1	8	**1.6**	**27**	**0.1**	**42**
(Bhattacharya and Saramaki, 2003) MNSPT design.						**1.81**	**29**	**0.16**	**41.2**

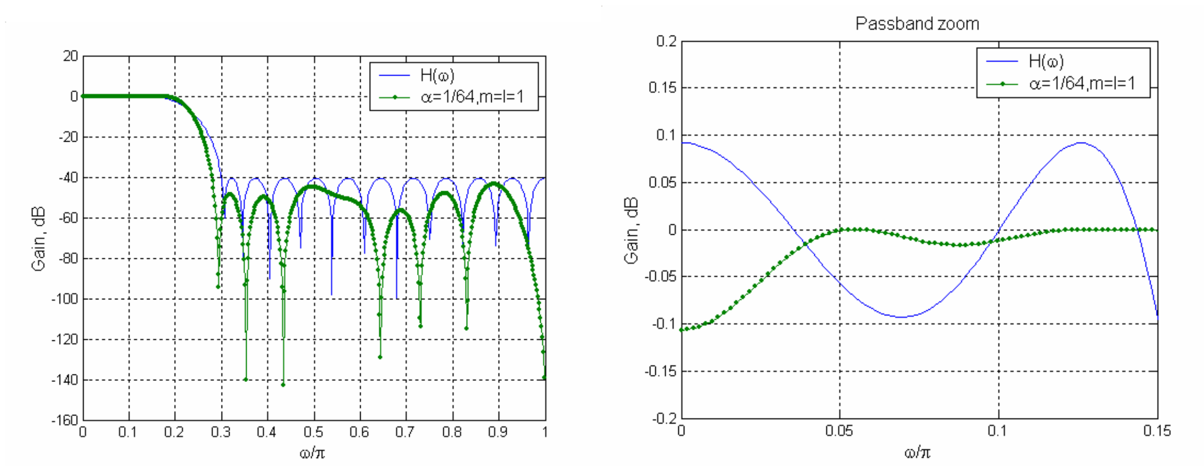


Fig. 8. Proposed revised design of the Filter 1

In the following we consider the Filters 2, 3 and 4 from (Bhattacharya and Saramaki, 2003) designed using the proposed method and the MNSPT design..

Filter 2: $\omega_p = 0.2\pi\ \omega_s = 0.3\pi, R_p = 0.5dB, As = 50dB.$ The results are summarized in Table 5. Note that the proposed design for α=1/64 is much better (See row 2 in Table V), than the MNSPT designs.

Table 5. Results for Filter 2

α	N_1	N_2	m	1	Nb_1	*Nbm*	*Nb*	*R* [dB]	*A*[dB]
1/32	17	5	2	3	8	1.6	62	0.3	50
1/64	23	6	1	2	11	**1.83**	**47**	**0.2**	**58**
1/128	35	9	1	1	16	1.77	51	0.05	51
(Bhattacharya and Saramaki, 2003) Initial MNSPT design.						**3.71**	**78**	**0.42**	**51**
(Bhattacharya and Saramaki, 2003) Revised MNSPT design (*Rp*=0.5 dB , *A*=55 dB).						**2.68**	**59**	**0.49**	**52.1**

Filter 3: $\omega_p = 0.1\pi\ \omega_s = 0.2\pi, R_p = 0.3dB, As = 60dB.$ The results are shown in Table 6. Best proposed design is obtained for α=1/32 and m=2 *and l*=3*,* and is much better than MNSPT design.

Table 6. Results for Filter 3

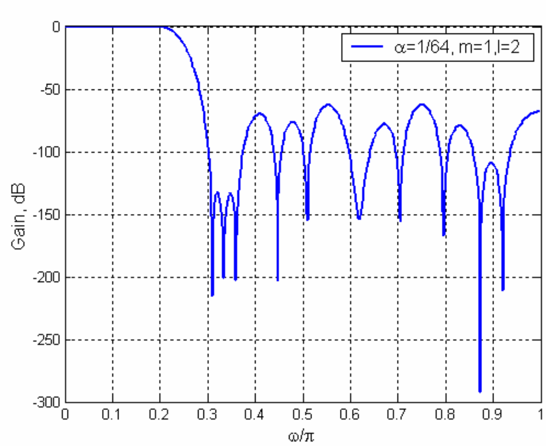
α	N_1	N_2	m	l	Nb_1	*Nbm*	*Nb*	*R* [dB]	*A*[dB]
1/32	20	3	2	3	4	**1.33**	**34**	**0.1**	**61**
1/64	28	6	1	2	9	1.5	39	0.12	62.2
1/128	38	8	1	2	14	1.75	59	0.07	72
(Bhattacharya and Saramaki, 2003) Inicial MNSPT design.						**3.96**	**107**	**0.275**	**60.3**
(Bhattacharya and Saramaki, 2003) Revised MNSPT design (*Rp*=0.3dB , *A*=75 dB).						**2.68**	**83**	**0.272**	**62.25**

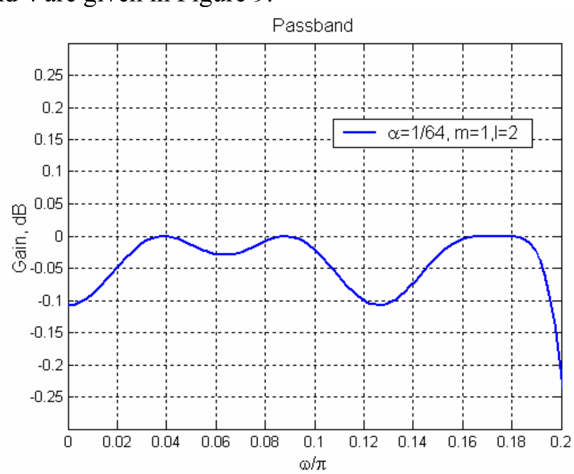
Filter 4: $\omega_p = 0.1\pi, \omega_s = 0.15\pi, R_p = 0.1dB, As = 50dB.$ The results are given in Table 7. The proposed design again exhibits better results than the initial and revised MNSPT designs.

Table 7. Results for Filter 4

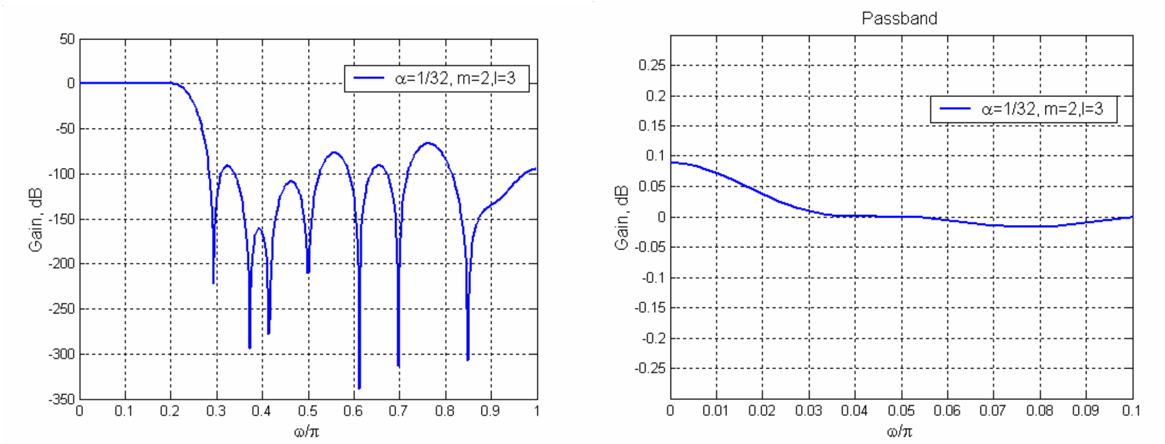
α	N_1	N_2	m	1	Nb_1	*Nbm*	*Nb*	*R* [dB]	*A*[dB]
1/64	39	6	3	3	9	**1.5**	**103**	**0.07**	**72**
(Bhattacharya and Saramaki, 2003) Initial MNSPT design.						**3.4**	**177**	**0.093**	**50.5**
(Bhattacharya and Saramaki, 2003) Revised MNSPT design (*Rp*=0.09 dB , *A*=60 dB).						**2.36**	**135**	**0.095**	**52.01**

The proposed designs shown in bold for Filters 2, 3 and 4 are given in Figure 9.

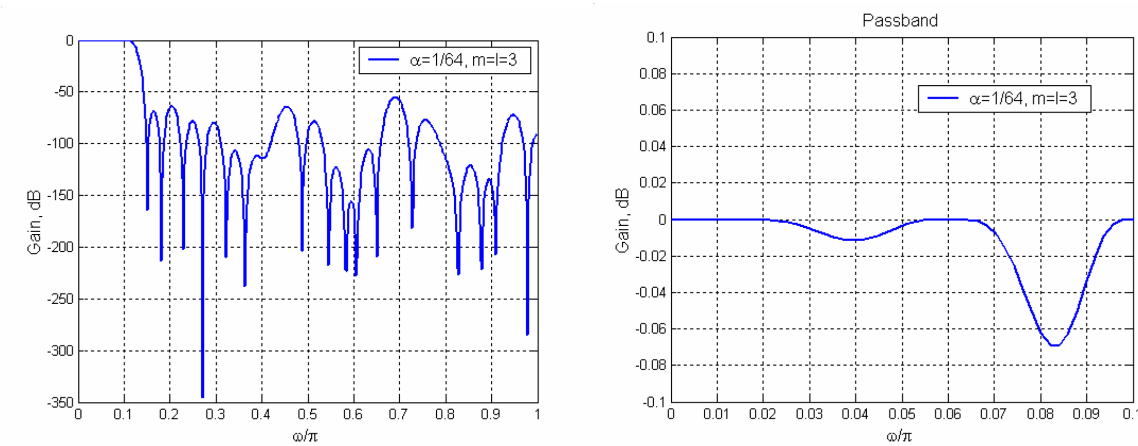




a. Filter 2



b. Filter 3



Filter 4

Fig. 9. Proposed design of Filters 2, 3 and 4

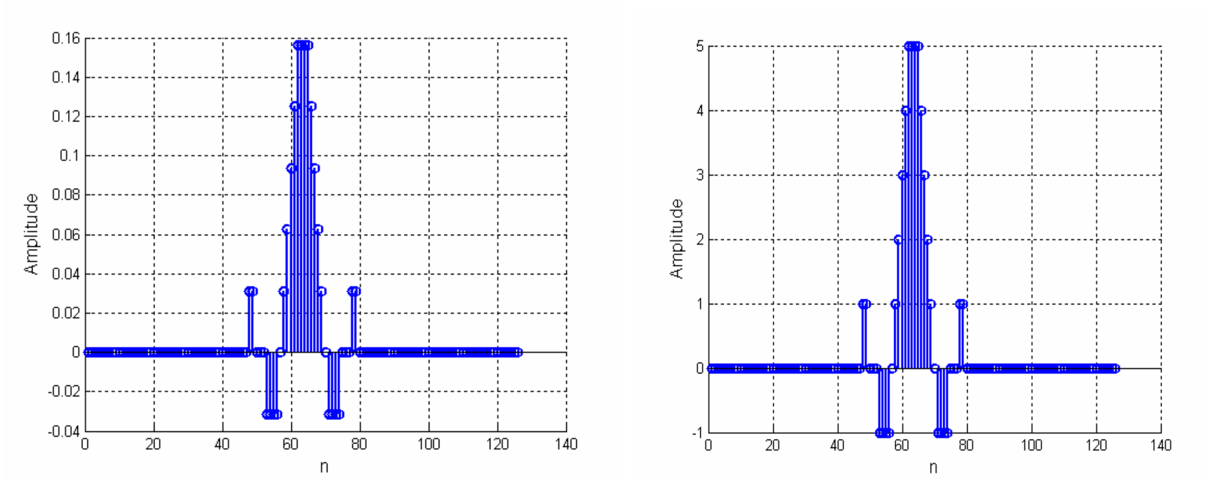
In the following we compare our design with the method (Bhattacharya and Saramaki, 2003). The normalized passband and the stopband frequencies are 0.15625 and 0.1875, respectively. The resulting multiplierless filters must have at least attenuation of 80 dB. To this end the infinite precision filter $hh(n)$ of an overall length of 123 and the stopband attenuation of 67.37dB is designed. In the next this filter is optimized along with the corresponding sharpening polynomial .

Using the rounded constant $r=2^{-5}= 0.313$ we have the rounded impulse response of the filter $hh(n)$, shown in Fig.11.a. The corresponding impulse response with integer coefficients (See Eq (1)) is demonstrated in Fig. 10.b.

The total length of rounded filter is 24 and there is 4 integer coefficients: 2, 3, 4, and 5, which require the total number of 6 bits.

Using the sharpening polynomial (m=1, l=1), the sharpened rounded filter requires 20 bits.

The cascade of four sharpened filters needs 80 bits and satisfies the required specification. Using the cascade of two sharpened rounded filters, where the parameters of the sharpening polynomial are m=1 and l=3, the specification is satisfied (Fig.11) but the total number of bits is less and equal to 64. In both cases the results are better than using optimization procedure (Bhattacharya and Saramaki, 2003).



a. b.

Fig.10. Rounded impulse responses for filter (Bhattacharya and Saramaki, 2003)

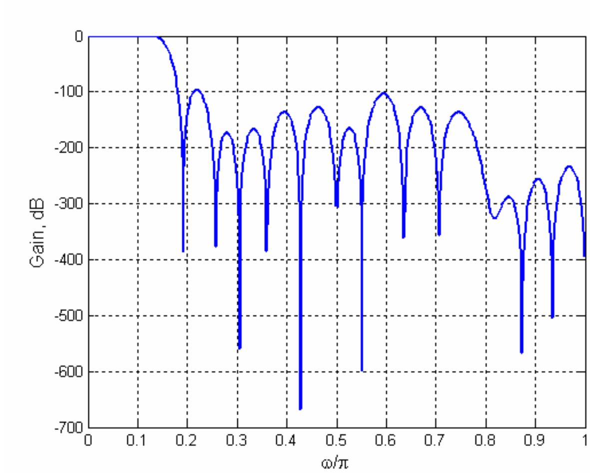


Fig. 11. The sharpened rounded filter, r=2^{-5}, m=1, l=3

6 Concluding Remarks

In general, optimization techniques usually used for multiplierless filter design are complex, can require long run times, and provide no performance guarantees (Koter at al., 2003). The main goal of our work was to propose simple efficient method for the design of multiplier-free FIR filters without optimization.

The method uses the rounding to the nearest integer of the coefficients of the equiripple filter which satisfies the desired specification. Considering that the integer coefficient multiplications can be accomplished with only shift-and-add operations, the rounded impulse response filter is multiplier-free.

The complexity of the rounded filter (the number of the sums and the number of integer multiplications) depends on the choice of the rounding constant. Higher values of the rounding constant lead to the less complexity of the rounded filter but also in a more distortion in the desired gain response. In the next step the sharpening technique is used to improve the magnitude characteristic and to satisfy the specification. In that way the overall filter is based on combining one simple filter with integer coefficients.

The parameters of the design are the rounding constant and the parameters of the sharpening polynomials. As illustrated in examples the magnitude characteristics can have very small ripples in the passband.

Different examples show that the total number of nonzero bits in the proposed design is less than in the corresponding MNSPT design, while the magnitude response exhibits better characteristics. The cost is the increase of the total number of additions and delays which depends on the complexity of the sharpening polynomials.

As demonstrated in examples, the same holds if the revised MNSPT design, (Bhattacharya and Saramaki, 2003), using more strict specification, is applied.

The proposed design uses the sharpened polynomials from (Kaiser and Hamming, 1997), and results in a better performances than the method based on simultaneous optimization of the subfilter and the corresponding sharpening polynomial coefficients.

Unlike to methods (Tai and Lin, 1992) and (Jovanovic at al., 2005) there is no restriction in the specification of the filter. This way can be designed both the narrowband and the wideband multiplierless filters. However, the method exhibits more efficient results if the order of the initial filter designed using Parks McClellan algorithm is less than 150. In that case the masking method (Lim, 1986) can be used to obtain less order component filters and to apply the proposed procedure for the model and masking filters.

7 References

1. **J. A Adams** and **A. N. Williamson, Jr**., "A new approach to FIR digital filters with fewer multipliers and reduced sensitivity," *IEEE Trans. Circuits and Systems*., vol. CAS-30, pp. 277-283, May 1983.
2. **A. Bartolo, B. D. Clymer, R. C. Burges,** and **J. P. Turnbull**, "An efficient method of FIR filtering based on impulse response rounding," *IEEE Trans. on Signal Processing*, vol. 46, No. 8, pp.2243-2248, August 1998.
3. **M. Bhattacharya** and **T. Saramaki**, "Some observations leading to multiplierless implementation of linear phase filters," *Proc. ICASSP 2003*, pp. II-517-II-520.
4. **T. S. Chang, Y. H. Chu** and **C.W. Jen**, "Low –power FIR filter realization with differential coefficients and inputs," *IEEE Trans on Circuits and Systems: Part II,* vol.47, pp.137-145, February 2000.
5. **M. Donadio**, " Lost knowledge refound: Sharpened FIR filters, " IEEE Signal Processing Magazine, pp.61-63, September, 2003.
6. **P. Gentili, F. Piaza** and **A. Uncini**, "Improved power-of-two sharpening filter design by genetic algorithm," Proc of *IEEE Conference on Acoustics and Signal Processing*, vol.3, pp.1375-1378, May 1996.
7. **R. Harnett,** and **G. F. Boudreaux-Bartels**, "Improved filter sharpening," *IEEE Trans on Signal Processing,* vol.43, No.12, pp. 2805-2809, Dec. 1995.
8. **G. Jovanovic Dolecek** and **S.K. Mitra**, "Design of FIR lowpass filters using stepped triangular approximation," *Proc. NORSIG 2002*, Norway, Oct. 2002.
9. **G. Jovanovic Dolecek** and **G. Espinosa**, "A method for FIR filter design based on quantization and PI technique," *Proc. ICCDCS 2000*, Cancun, Mexico, pp. S-35-1-S-35-6, May 2000.
10. **G. Jovanovic-Dolecek, M. M-Alvarez** and **M. Martinez**, "One simple method for the design of multiplierless FIR filters," Journal of Applied Research and Technology Vol. 3 No. 2, pp.125-138, August 2005.
11. **J.F. Kaiser** and **R.W. Hamming**, "Sharpening the response of a symmetric nonrecursive filter by multiple use of the same filter," *IEEE Trans., Acoust. Speech, Signal Processing*, vol. 45, pp. 457-467, February 1997.
12. **K. A. Kotteri, A. E. Bell,** and **J. E. Carletta**, "Quantized FIR Filter Design Using Compensating Zeros," *IEEE Signal Processing Magazine*, Nov. 2003, pp. 60-67.
13. **Y. C. Lim**, "Frequency-response masking approach for the synthesis of sharp linear phase digital filters," IEEE Transactions on Circuits and Systems, vol.CAS 33, pp.357-364, April 1986.
14. **Y. C. Lim, J. B. Evans,** and **B. Liu**, "Decomposition of binary integers into signed powers-of-two terms," *IEEE Trans. Circuits and Sysems*, vol. CAS-38, pp. 667-672, June 1991.
15. **Y. C. Lim, R. Yang,** and **D. Li**, "Signed Power-of-Two Term Allocation Scheme for the Design of Digital Filters,", *IEEE Trans. on Circuits and Systems- II: Analog and Digital Signal Processing*, vol.46, No5, May 1999, pp.577-584.

16. **Y. C. Lim** and **B. Liu**, “Design of cascade form FIR filters with discrete valued coefficients,” *IEEE Trans., Acoust. Speech, Signal Processing*, vol. ASSP-36, pp. 1735-1739. Nov. 1988.
17. **S. K. Mitra**, *Digital Signal Processing - A Computer-Based Approach*, Third Edition, McGraw-Hill Inc., New York, 2006.
18. **Y. L. Tai,** and **T.P. Lin**, “ Design of multiplierless FIR filters by multiple use of the same filter, Electronic letters, vol.28, pp.122-123, 1992.
19. **A. P.Vinod, E. M. K. Lai, A. B. Premkumar** and **C. T. Lau**, “FIR filter implementation by efficient sharing of horizontal and vertical common subexpressions,” *IEE Electronic Letters*, vol.39. No.2, pp. 251-253, January 2003.
20. **P. P. Vaidanathan** and **G. Beitman**, “On prefilters for digital filter design,” *IEEE Trans. on Circuitsand Systems*, vol. CAS-32, pp.494-499, May 1985.
21. **A. P. Winod, C. H. Chang, P.K. Meher,** and **A. Singla**, “Low power FIR filter realization using minimal difference coefficients: Part I- Complexity analysis,” Proc of the *IEEE Conference APCCAS 2006*, Singapore, pp.1549-1552.
22. **A. P. Winod, C. H. Chang, P.K. Meher,** and **A. Singla**, “Low power FIR filter realization using minimal difference coefficients: Part II- Algorithm,” Proc of the *IEEE Conference APCCAS 2006*, Singapore, pp.1553-1556.
23. **Wu-Sheng Lu**, “Digital Filter Design: Global Solutions via Polynomial Optimization,” Proc of the *IEEE Conference APCCAS 2006*, Singapore, pp.49-52.

Acknowledgement

This work is supported by CONACYT grant no. 49640.

Gordana Jovanovic Dolecek *received a PhD degree from the Faculty of Electrical Engineering, University of Sarajevo. She was professor at the Faculty of Electrical Engineering, University of Sarajevo until 1993, and 1993-1995 she was with the Institute Mihailo Pupin, Belgrade. In 1995 she joined Institute INAOE, Department for Electronics, Puebla, Mexico, where she works as a professor and researcher. During 2001-2002 and 2006 she was at Department of Electrical & Computer Engineering, University of California, Santa Barbara, as visiting researcher. Her research interests include digital signal processing and digital communications.*

Sanjit K. Mitra *received the M.S. and Ph.D. degrees in Electrical Engineering from the University of California, Berkeley, in 1960 and 1962, respectively. He has been a Professor of Electrical and Computer Engineering at the University of California, Santa Barbara since 1977, where he served as Chairman of the Department from July 1979 to June 1982. He has published over 600 papers in signal and image processing, twelve books, and holds five patents. He has been awardedhonorary doctorate degrees from the Tampere University of Technology, Finland, and the 'Politehnica' University of Bucharest, Romania.*